NEAR-SURFACE BAYESIAN FIRST-ARRIVAL TOMOGRAPHY WITH UNCERTAINTY USING NATURAL NEIGHBOR INTERPOLATION

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Summary

Various velocity parameterizations are used in Bayesian first-arrival tomography. We conduct a short review of the existing approaches and suggest the natural neighbor interpolation as a viable alternative. This parameterization possesses numerous useful properties. It provides naturally smooth models, which is particularly suitable for a refraction setting. It does not need any specific treatment at model boundaries, and, finally, does not need any additional parameters apart from velocities defined on a set of nodes. We compare this parameterization with a more conventional linear barycentric approach on a synthetic near-surface seismic dataset. The comparison shows that natural neighbor-based tomography results in a more accurate estimation of seismic velocity inside the near-surface low-velocity anomaly and provides a lower estimate of velocity uncertainty in the whole model.

Near-surface Bayesian first-arrival tomography with uncertainty using natural neighbor interpolation

Introduction

Bayesian approach to first-break seismic tomography has been widely used for velocity uncertainty estimation. Several different methods exist to provide the probabilistic solution of the tomographic problem: Hamiltonian Monte Carlo (Fichtner et al., 2019), variational inference methods (Zhang and Curtis, 2020), and reversible-jump Markov Chain Monte Carlo (rj-MCMC) (Bodin et al., 2012; Egorov et al., 2020). We select the last method due to its many published applications to field datasets (Bodin and Sambridge, 2009; Galetti et al., 2015; Ryberg and Haberland, 2018). In this approach, the velocities are defined on a set of nodes. The positions of the nodes and the velocity values are treated as unknowns. The velocity in the whole domain is estimated from the node velocities using the parameterization of choice. The original formulation of the Bayesian tomography with rj-MCMC uses Voronoi tessellation (i.e. nearest-neighbor interpolation) as a parameterization method. With this parameterization, the velocity field is piecewise-constant. A few other parameterizations are suggested for Bayesian tomography. For example, Belhadj et al. (2018) use Johnson-Mehl tessellation, which also results in piecewise-constant velocities but with more complex shapes of cell boundaries. The authors also propose a parameterization with Gaussian kernels, which results in smoother models. However, both Johnson-Mehl and Gaussian parameterizations require extra parameters per node, such as cell implantation times for the former one and widths of Gaussian kernels for the later one (in addition to usual node locations and velocities/kernel amplitudes). Hawkins et al. (2019) examine two more parameterizations based on Delaunay triangulation: linear barycentric interpolation and Clough-Tocher interpolation. These parameterizations provide smooth models and reduce the parameterization-induced error when the actual velocity model is smooth, which the authors demonstrate.

We suggest a new parameterization utilizing the natural neighbor interpolation (Sibson, 1981). Natural neighbor interpolation is based on Voronoi tessellation of the set of nodes. The velocity value at any point is computed as a weighted sum of the neighboring nodes, in contrast to the more well-known nearest neighbor method, where the velocity at any point is set equal to the nearest node's velocity. Natural neighbor interpolation has numerous advantages when applied as a parameterization in Bayesian tomography. First, it provides smooth models. Linear barycentric parameterization was found more accurate in a refraction setting with smooth true model by Ryberg and Haberland (2018). The natural neighbor method produces even smoother models, which are continuously differentiable everywhere except the nodes. Second, the natural neighbor interpolation provides no overshoots or undershoots (in contrast to some variants of the Clough-Tocher interpolation method, for example), which means that the velocities everywhere in the model are always between the minimum and maximum velocities at the nodes. The range of velocities at the nodes, in turn, is controlled by the priors. Hence, no unexpected velocity values appear in the models, and no extra model clipping is required to enforce the priors in the domain. Third, in contrast to Johnson-Mehl and Gaussian parameterizations, the natural neighbor interpolation does not require any additional parameters. As a consequence of these properties, the natural neighbor parameterization can be considered a viable alternative for Bayesian tomography. We demonstrate the advantages of this parameterization on a synthetic example designed to replicate a near-surface exploration seismic survey to detect near-surface low-velocity anomalies.

Method

We use an rj-MCMC Bayesian tomography method, as explained by Bodin et al. (2012). Velocities are defined on a set of nodes, and the velocities between the nodes are constructed by an interpolation method of choice. We use an eikonal solver for the computation of traveltimes that needs the velocities defined on a regular grid. To interpolate the node velocities onto this regular grid, we apply the effective discrete natural neighbor 'scatter approach' computation scheme described by Park et al. (2006). This implementation of the natural neighbor method is very efficient in computation time; however, we choose it mainly due to the ease of implementation. We expect that for the considered 2D problem with a relatively low number of nodes (less than one thousand), more simple geometric approaches to the natural neighbor interpolation problem may also be feasible.

In Figure 1, we show a comparison of different interpolation methods used as parameterizations for rj-MCMC tomography. We generate a set of nodes with defined velocities and interpolate these velocities onto the regular grid by the nearest neighbor, natural neighbor, and linear barycentric interpolation (Figure 1). It can be observed that the natural neighbor method provides a smoother model. Note that the linear barycentric parameterization requires special treatment of the model edges. In this method, the interpolated function is computed only in the convex hull of the nodes. We apply the same treatment as Ryberg and Haberland (2018) by adding four artificial points at the corners of the model with the velocity values equal to the nearest node's velocities. Still, this creates large triangles along the model edges (see arrow in Figure 1). This may influence the refraction tomography results, as the sources and receivers are located along the model's upper edge. The velocities along the edge heavily influence the traveltimes. This problem, however, may be solved by adding more artificial points to the model boundaries. Note that the nearest and natural neighbor methods do not require any special treatment of model boundaries.

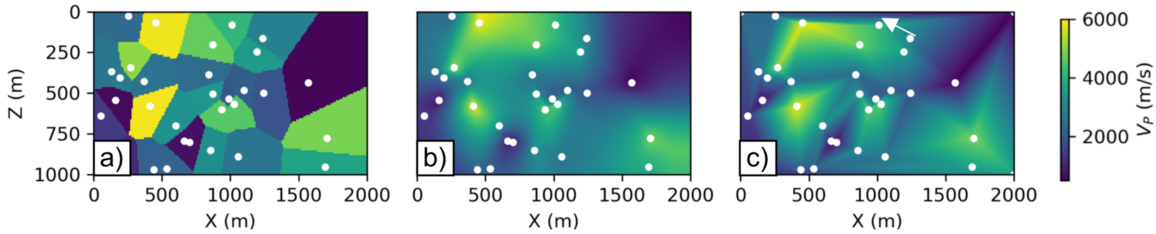

***Figure 1** A comparison of the nearest neighbor interpolation (a), natural neighbor interpolation (b), and barycentric linear interpolation (c) for the same set of nodes.*

Example

We test the new parameterization on the model shown in Figure 2a. Subsurface is characterized by a relatively smooth model containing low-velocity anomalies in the near-surface. We perform full-waveform finite-difference modeling of surface seismic acquisition and pick the first arrivals (Figure 2b). We then invert these manually picked first-arrival traveltimes. The source spacing is 100 m, the receiver spacing is 10 m. To simulate a realistic seismic survey, we model a moving spread with minimum and maximum offsets equal to 100 m and 1400 m, respectively. The maximum offset was picked to avoid the shingling of first breaks at large offsets caused by a low-velocity layer in the model. The minimum offset choice is motivated by the fact that picking of first arrivals on land vibroseis data at short offsets may be complicated by intense non-causal correlation-related artifacts and harmonic noise.

For comparison, we run the rj-MCMC tomography with two parameterizations: linear barycentric and natural neighbor. For both methods, we run 90 chains until each of the chains accepts 30000 samples. We then discard the first 28000 samples as burn-in, decimate the chains by selecting every 10th sample and compute the mean velocity models (Figure 2c-d) and the standard deviations (Figure 2e-f) for two parameterizations. Note that the velocity prior distributions are uniform with bounds equal to 700 m/s and 8000 m/s. The mean velocity models are comparable. However, it can be argued that the low-velocity anomaly at x = 3500 m is more pronounced in the natural neighbor model. The standard deviation of the velocity is significantly lower in the natural neighbor parameterization. For example, inside the low-velocity anomaly (identified as a black point in Figure 1a), the standard deviations are equal to 160 m/s and 60 m/s for linear barycentric and natural neighbor parameterizations, respectively. Such behavior may be interpreted as a lower 'parameterization error' for the natural neighbor (Hawkins et al., 2019) caused by the treatment of the model edges or because the chains have not fully converged for the linear parameterization. However, we could not obtain a significantly different standard deviation for linear parameterization after running the chains for a twice longer time. Figure 2g-j shows that the stochastic models from natural neighbor parameterization resemble the true model more closely in terms of smoothness. As the traditional Bayesian tomography does not permit the user to constrain the models' structural properties and smoothness directly, we see this as a useful property.

Convergence curves for the chains for two parameterizations are displayed in Figure 3a. The Figure suggests more stable (but not necessarily quicker) convergence for natural neighbor tomography.

However, this cannot be considered as a benchmark – the sampling parameters, such as proposal distributions (Bodin and Sambridge, 2009), were kept constant for both tests. Hence, they may not be optimal for each of the methods.

In Figure 3b, we display the posterior velocity distributions inside the low-velocity anomaly identified by the two parameterizations and compare them to the true value of velocity. The exact spatial location for these distributions' computation is shown as a black point in Figure 2a. Both parameterizations overestimate the velocity, however, the distribution provided by the natural neighbor parameterization is closer to the true value. It can also be observed that the linear parameterization results in a higher number of low-velocity outliers compared to the natural neighbor result. This may be related to the treatment of velocity models in linear barycentric parameterization near the model edges and may contribute to the increased standard deviations described above.

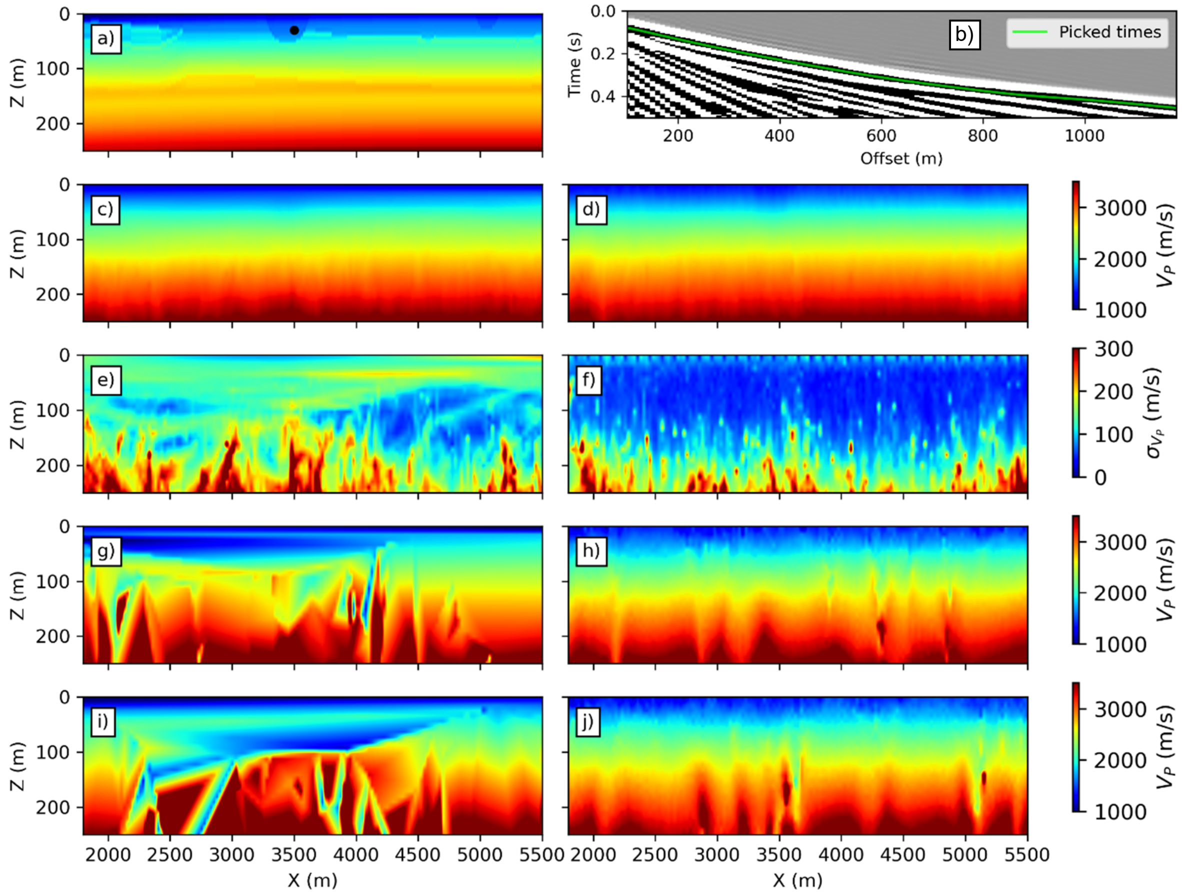

Figure 2 *True subsurface model (a), seismic gather with example picks (b), mean velocity models (c-d), standard deviations (e-f), and examples of stochastic models from the ends of the chains (g-j) for barycentric linear parameterization (c, e, g, i) and natural neighbor parameterization (d, f, h, j).*

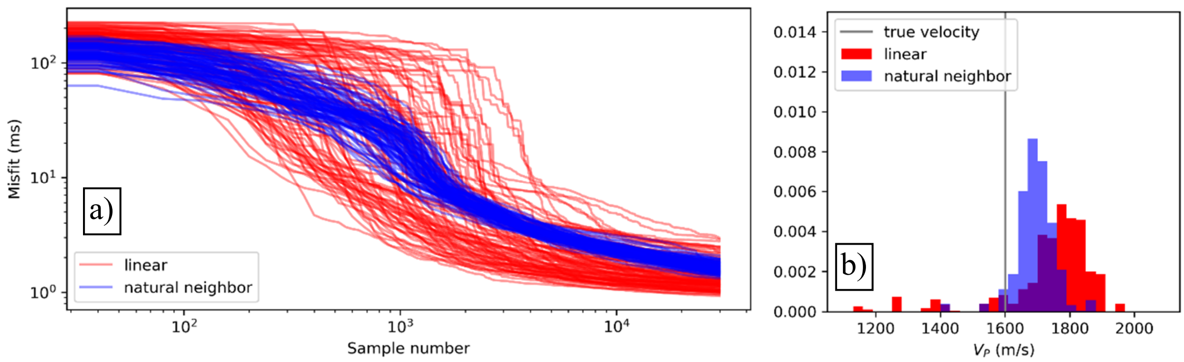

Figure 3 *Misfit curves (a) and (b) posterior distributions of velocity inside the low-velocity anomaly (b) for the linear barycentric and natural neighbor parameterizations. The velocity distributions were computed for the spatial location shown as a black point in Figure 2a.*

Conclusions

Natural neighbor shows to be a viable option as a parameterization for transdimensional Bayesian tomography. It does not require additional parameters and provides naturally smooth stochastic models with no overshoots and undershoots. We compare it to the linear barycentric parameterization using a synthetic dataset simulating exploration-scale near-surface acquisition with the application of refraction tomography. Both parameterizations successfully estimate the velocities. In the demonstrated model example, the velocity uncertainty estimated with the natural neighbor parameterization is lower than with barycentric linear parameterization. The stochastic models generated by the natural neighbor algorithm resemble the true model more closely. Also, natural neighbor parameterization provides a more accurate estimate of velocity inside the near-surface low-velocity anomaly.

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