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Combining Discontinuous Galerkin with Finite Differences to Simulate Seismic Waves in Presence of Free-surface Topograph

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SUMMARY

We present an original algorithm for numerical simulation of seismic wave propagation in models with complex topography along a free surface and other near-surface complexities. Conventional methods such as finite-difference seismic modeling either generate many artifacts or require an extremely fine discretization grid that is computationally expensive. The new approach combines discontinuous Galerkin in the complex near-surface part of the model with conventional rectangular finite-difference modeling for the deeper, less complex part. Using a triangular mesh in the near surface allows more accurate description of topography without artifacts or excessive discretization, while additional computation costs are minimized by only applying it to shallow portions of the subsurface models.

Introduction

Simulation of seismic wave propagation is typically performed by various finite-difference (FD) schemes, because these methods combine high efficiency, low computational cost, universality and acceptable accuracy (Virieux *et al.*, 2011). However, the presence of high-contrast interfaces and especially a free surface may seriously reduce the accuracy of simulations. This is mostly associated with the stair-step approximation of dipping interfaces caused by rectangular gridding used for FD. This stair-step approximation leads to diffractions, which can be particularly strong when surface waves interact with the interfaces. In this case the converted body waves may be as strong as those emitted by the main source, i.e. secondary sources appear in the model. This effect is clearly seen in Figure 1.

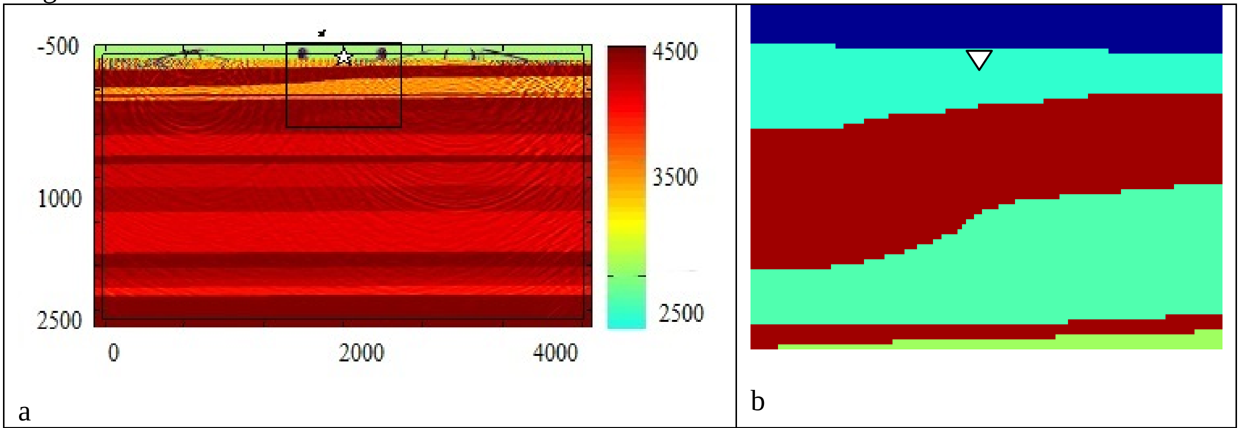

Figure 1 *(a) A model with low-dip angle of the free surface and associated wavefield, generated by the finite differences; (b) zoomed upper part (black rectangle in a) of the model indicates presence of two "steps" in the approximation of the free surface. White star indicates source position.*

In contrast, finite-element (FE) methods can be used for simulation of wave propagation on unstructured tetrahedral (triangular) grids. In this case, interfaces are approximated with higher smoothness and diffractions become negligible. However, use of FE and in particular the discontinuous Galerkin (DG) method (Etienne et al. 2010) considered in this paper, significantly increases the computational intensity of the simulation if used for the entire model. In addition, grid generation becomes time-consuming and tedious.

This paper presents a hybrid algorithm that combines DG with standard staggered-grid scheme (SSGS) for finite differences (Virieux, 1986). In our case DG is only used in the near surface thus allowing proper approximation and treatment of the free surface whereas the deeper portion of the model is computed with SSGS preserving the low computational cost of the simulation.

Description of the algorithm

Let us assume a free surface $z_{fs} = z_{fs}(x)$, introduce an artificial interface (it has no connection with any of the physical interfaces) $z = z_{DG}$ and generate a triangular grid inside this area (Figure 2). Formulation of the DG method (Etienne *et al.*, 2010) for the velocity-stress divergence-free formulation of elastic wave equation becomes

$$\int_{V_k}\left(\begin{pmatrix}\rho I & 0\\ 0 & S\end{pmatrix}\frac{\partial}{\partial t}\begin{pmatrix}u\\ \sigma\end{pmatrix},\begin{pmatrix}\varphi\\ \psi\end{pmatrix}\right)dv+\frac{1}{2}\sum_{j=1}^{2}\int_{V_k}\left(\begin{pmatrix}u\\ \sigma\end{pmatrix},\begin{pmatrix}0 & B_j\\ B_j^* & 0\end{pmatrix}\frac{\partial}{\partial x_j}\begin{pmatrix}\varphi\\ \psi\end{pmatrix}\right)dv-\frac{1}{2}\sum_{j=1}^{2}\int_{V_k}\left(\begin{pmatrix}0 & B_j\\ B_j^* & 0\end{pmatrix}\frac{\partial}{\partial x_j}\begin{pmatrix}u\\ \sigma\end{pmatrix},\begin{pmatrix}\varphi\\ \psi\end{pmatrix}\right)dv-$$

$$-\frac{1}{2}\sum_m\int_{S_{km}}\left(\begin{Bmatrix}u\\ \sigma\end{Bmatrix},\left[\begin{pmatrix}0 & B_{km}\\ B_{km}^* & 0\end{pmatrix}\begin{pmatrix}\varphi\\ \psi\end{pmatrix}\right]\right)ds+\frac{1}{2}\sum_m\int_{S_{km}}\left(\left[\begin{pmatrix}0 & B_{km}\\ B_{km}^* & 0\end{pmatrix}\begin{pmatrix}u\\ \sigma\end{pmatrix}\right],\begin{Bmatrix}\varphi\\ \psi\end{Bmatrix}\right)ds=\begin{pmatrix}F_u\\ F_\sigma\end{pmatrix}$$

where u and σ are the velocity vector and the stress tensor respectively, whereas ρ is the mass density, S is the elastic compliance tensor, matrices B_j are composed of zeroes and ones and can be found, for example in Lisitsa et al. (2012). Functions φ and ψ are trial functions, which we consider to be polynomials of degree l within the cell V_k and zeros otherwise. S_{km} is the edge of the cell shared with the m[th] neighbour (see Figure 3), $\{f\}$ – denotes the mean of the f at the interface, $[f]$ - means a jump across the interface and, $B_{km} = \sum_{j=1}^{2} n_{km}^{j} B_j$, where $\vec{n}_{km}$ is the vector of the outer normal. Further discretization in time is applied by a leap-frog scheme. In order to couple the DG with the SSGS we suggest introducing a transition zone $z_{DG} < z < z_R$ where properties of the two methods are combined (Figure 2). In particular we use the DG on a regular rectangular grid with the trial functions that are constants (polynomials of degree 0). As it follows, for example, from Ainsworth et al. (2006), in this case the DG coincides with the finite-difference scheme on conventional (non-staggered) grid. This means that the problem can be split into two independent ones:

- Coupling of DG on triangular grid with DG on rectangular grid at the interface $z = z_{DG}$;
- Coupling of non-staggered grid scheme with the SSGS, at the interface $z = z_R$.

The first problem can be solved straightforwardly because an essential property of the DG is a possibility of using arbitrary nonconforming grids and choosing the trial and basis functions locally, which is also known as hp-adaptivity. The solution of the second problem is based on the specific properties of the velocity-stress formulation of the finite-difference approximation to the elastic wave equation. The non-staggered grid scheme possesses artificial "plus-minus" modes, but the SSGS does not. Thus the formulae to couple the two schemes are designed in such a way that we let the true modes propagate through the artificial interface $z = z_R$ with as low a reflection as possible, but prevent propagation of the artificial modes. The approach is analogous to coupling of Lebedev scheme and SSGS, presented by Lisitsa *et al*. (2012). As a result, the artificial reflections (the main numerical error) caused by the coupling of DG and standard staggered grid schemes are as low as $10^{-3} - 10^{-4}$ of the incident wave, which is an acceptable level for seismic simulation.

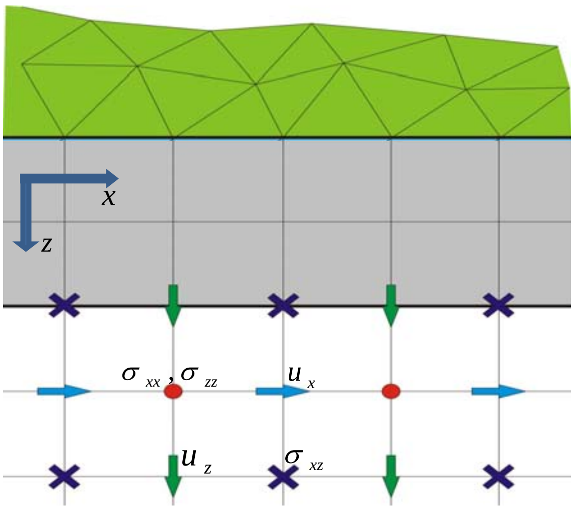

Figure 2 *A sketch of the grids used to in hybrid algorithm. Different markers at the bottom represent the SSGS area, grey domain is the transition zone, green area is the DG zone.*

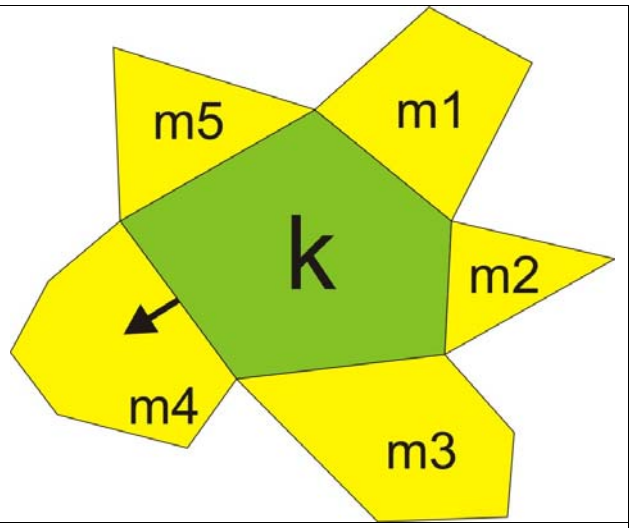

Figure 3 *A representation of a grid cell and its neighbours for the DG.*

Numerical experiments

To demonstrate capabilities of our hybrid approach we present two numerical experiments. The first comprises a 2D synthetic model containing significant topography variations of the free-surface interface as is often the case for land seismic. The subsurface is represented by homogeneous material with $V_P = 4500m/s$, $V_S = 3000m/s$, and $\rho = 2000kg/m^3$. The model is 8 km long and 4 km deep whereas topography is represented as a sine function with 3.5 periods per 8 km (Figure 4). A volumetric source is located 5 m below the surface at $x = 4000$ m. We compare simulations done with pure SSGS, pure DG algorithm and our hybrid approach. The spatial discretization was chosen to ensure 30 degrees of freedom per wavelength for DG and FD algorithms. Time stepping was also equal. Since DG (with polynomials of first degree) performs approximately twice as many floating-point operations, the runtime of the DG simulation was twice as long compared to the FD modelling. In the hybrid approach, the DG zone occupied around 30% of the model but delivered results of similar accuracy as a pure DG scheme with triangulation of the entire model with a runtime that was only 35% slower compared to pure finite differences. Figure 5 shows snapshots simulated by the hybrid approach and the SSGS. We observe that that propagation of the body waves and direct Rayleigh waves is modelled equally well, however finite differences show significant numerical artifacts/diffractions caused by discretization of dipping interfaces and along topography. The same effect is also observable in seismograms (Figure 6).

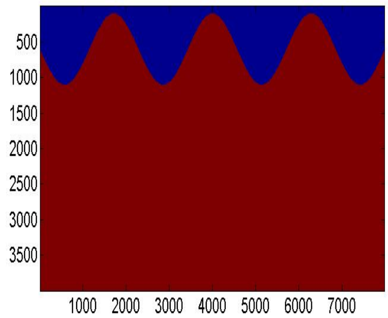

__Figure 4__ A sketch of the homogeneous model for the first experiment.

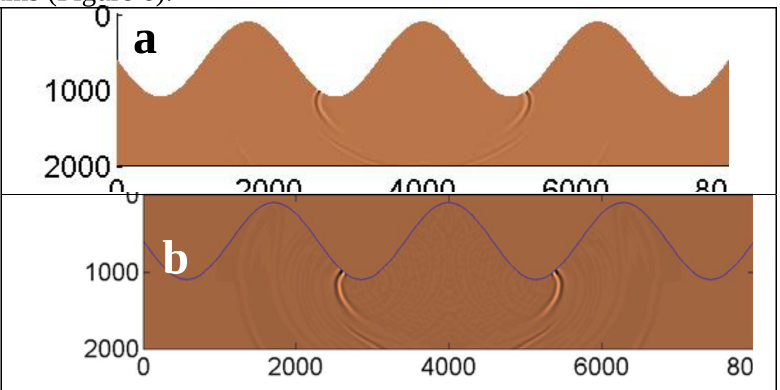

__Figure 5__ Snapshots of the vertical component of the displacement vector computed by the hybrid algorithm (a) and finite differences (b).

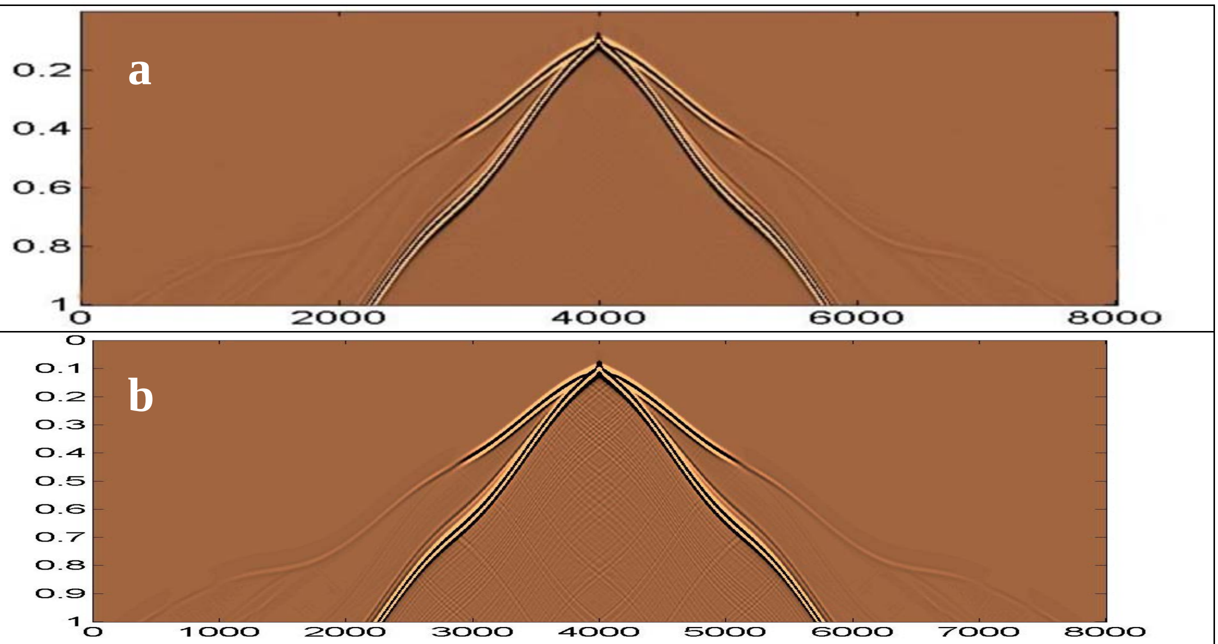

***Figure 6** Vertical components of the displacement velocity computed by the hybrid algorithm (a) and finite differences (b).*

The second test is conducted for a more complex "realistic" 2D elastic model from Eastern Siberia (Figure 7). A volumetric source was located at $x = 4125m$, $z = 110m$, ten meters below the free surface. The seismograms generated by the hybrid algorithm are shown in Figure 8. As before, we observe clear Rayleigh waves with no diffractions from stair stepping in the grid, which were observed on finite difference modelling with the same model (not shown here).

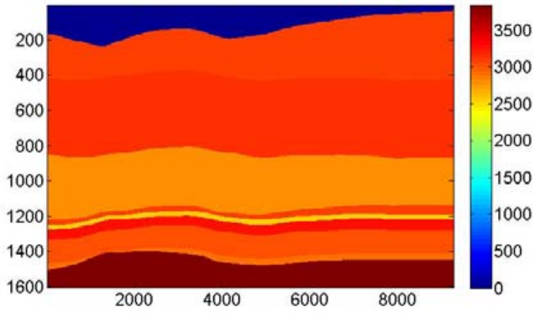

Figure 7 *Shear-wave velocity model used in the second test (blue represents air above topography).*

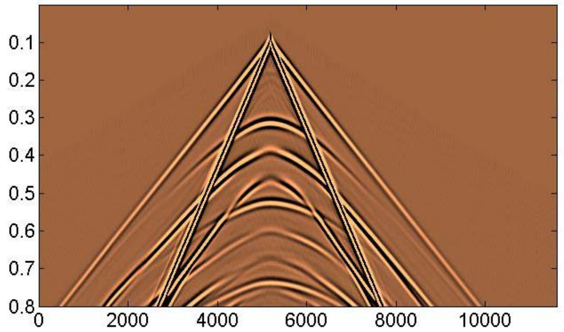

Figure 8 *Vertical displacement from a point explosive source recorded at the free surface for second test.*

Conclusions

We present a hybrid algorithm for numerical simulation of seismic wave propagation in models with complex topography including a free surface. The new approach is based on the combination of the discontinuous Galerkin method using unstructured triangular (tetrahedral) meshes overlying a regular finite difference grid. The DG method accurately handles complex topography, naturally allowing for local increases of the accuracy by using the hp-adaptivity technique. However DG is computationally intensive compared with finite differences and therefore it is only used in a shallow near-surface part of the model. Wave propagation in the deeper part of the model is simulated by the highly efficient standard staggered grid finite-difference scheme, which improves computational efficiency. As a result our approach preserves the high accuracy of DG and efficiency of the FD simulation of seismic wave propagation. The current version of the algorithm targets complex topography of the free surface. The same approach could be extended to handle complex bathymetry and rugose salt-sediment interfaces.

Acknowledgements

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References

Ainsworth, M., P. Monk, and W. Muniz [2006] Dispersive and Dissipative Properties of Discontinuous Galerkin Finite Element Methods for the Second-Order Wave Equation. *Journal of Scientific Computing*, **27**(1), 5-40.
Etienne, V., E. Chaljub, J. Virieux, and N. Glinsky [2010] An hpadaptive discontinuous Galerkin finite-element method for 3D elastic wave modelling. *Geophysical Journal International* **183**, 941-962.
Lisitsa, V., V. Tcheverda, and D. Vishnevsky [2012] Numerical simulation of seismic waves in models with anisotropic formations: coupling Virieux and Lebedev finite-difference schemes. *Computational Geosciences*, **16**(4), 1135-1152.
Virieux, J. [1986] P-SV wave propagation in heterogeneous media: Velocity - stress finite-difference method. *Geophysics*, **51**(4), 889-901.
Virieux, J., H. Calandra, and R.-É. Plessix [2011] A review of the spectral, pseudo-spectral, finite-difference and finite-element modelling techniques for geophysical imaging. *Geophysical Prospecting*, **59**(5), 794-813.