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Effect of Free-surface Related Multiples on Near Surface Velocity Reconstruction with Acoustic Frequency Domain FWI

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SUMMARY

Singular value decomposition (SVD) has proved to be a very effective tool for analysis of inverse problems. In this paper we present a methodology for iterative SVD in acoustic 2D full-waveform inversion (FWI). Using our approach, decomposition may be done for any complex 2D background velocity models of any size. As an example, we analyze two particular inversion scenarios using a synthetic model with complex near surface. We study the impact of surface-related multiples on reliability of near-surface velocity model reconstruction in acoustic frequency-domain FWI. We show that while using low frequencies, inversion scenarios without free-surface multiples may provide better resolution provided an optimal regularization parameter is chosen. Scenarios with surface-related multiples provide less well resolved results but are more stable with respect to noise and the choice of regularization.

Introduction

In arid environments the near surface represents a major obstacle to successful seismic imaging (Robinson and Al-Husseini, 1982). Full-waveform inversion (FWI) holds the promise to deliver a more detailed near surface model and enable imaging improvements. In practice, results from nonlinear FWI depend on many factors such as whether we use accurate physics, data quality, accuracy of initial model and the type and level of regularization. It is often difficult to distinguish the impact of different factors on a final result. Many expensive test-and-trial runs may be required in order to verify which scenario is preferable in a given situation and what is a fundamental reason behind. Linearized inverse problems are much more suitable for preliminary analysis of inversion scenarios because basic relations between the model parameters, parameters of inversion procedure and parameters of the observation system are simpler and better understood. In many cases, nonlinear FWI may be considered as a sequence of linearized problems (due to the use of the Newton-type minimization techniques), so by understanding the properties of the simple linear steps one can make important conclusions about nonlinear procedure as a whole.
In this paper, we present a methodology that can be used to analyze different inversion scenarios of acoustic FWI. The method is based on iterative singular value decomposition (SVD) of the linearized forward map which is considered in the temporal frequency domain. Such kind of analysis may be done for any complex 2D background velocity models of any size. As an example, we study the impact of surface related (SR) multiples on the reliability of near surface (macro) velocity model reconstruction. This question is of essence for FWI applications on land.

Method

In this study we are going to analyze basic properties of linearized FWI. If acquisition parameters and a reference model $\vec{m}_0$ are fixed, inversion may be written as a linear system: $L\vec{m} = \vec{d}$, where L is a linear forward map, $\vec{d}$ represents the residual data vector, and $\vec{m}$ is the unknown model perturbation within the target area. We consider a composite linear system (Silvestrov *et al.*, 2013):

$$L: \quad \begin{bmatrix} \hat{L}(\omega_1) \\ \vdots \\ \hat{L}(\omega_{N_f}) \end{bmatrix} \vec{m} = \begin{pmatrix} \vec{d}(\omega_1) \\ \vdots \\ \vec{d}(\omega_{N_f}) \end{pmatrix}, \qquad (1)$$

where each block $\hat{L}(m_0, \omega_j)$ is a linearized forward map corresponding to single frequency ω_j and reference model $\vec{m}_0$. Using the Born approximation for the acoustic case we define it as:

$$\hat{L}_j < m(\vec{x}) >= \omega_j^2 F(\omega_j) \int_\Omega m(\vec{x}) G(\vec{x}_S, \vec{x}; \omega_j) \cdot G(\vec{x}, \vec{x}_R; \omega_j) d\vec{x} \,,$$

where $G(\vec{x}, \vec{y}; \omega_j)$ is a Green function in the reference model, $F(\omega_j)$ describes spectrum of the source wavelet.
In order to analyze the basic properties of linearized FWI (Equation 1) such as resolution, stability with respect to noise, one needs to evaluate a singular value decomposition of the composite matrix of the system (Menke, 1994). To achieve this in practice we have to take into account following consideraions: **1)** for an arbitrary background model it is a computationally expensive process to construct the system (1) explicitly because it requires large number of wavefield simulations (proportional to the number of grid points in the target area); **2)** even in the 2D case, the entire size of the problem may be huge. So first of all we reduce the size of the problem. Premultiplying by the adjoint matrix L^* leads to the normal system of linear equations:

$$L^* L\vec{m} = L^*\vec{d} \Rightarrow \left[\sum_j \hat{L}^*(\omega_j) \hat{L}(\omega_j) \right] \vec{m} = \sum_j \hat{L}^*(\omega_j) \vec{d}(\omega_j) \,. \qquad (2)$$

Further we will consider a matrix $A \equiv real[L^* L]$ and perform its eigen-decomposition iteratively (Osypov *et. al.* 2008). We use a matrix-free implementation. In our case, iterative eigen-decomposition procedure needs to be provided with two routines: **i)** matrix-by-vector product, $L\, v_{IN}$, and **ii)** conjugate matrix-by-vector product, $L^{*} d_{IN}$. Both procedures may be described in terms of wavefield simulation which is governed by a Helmholtz equation. For each frequency, LU decomposition of a corresponding Helmholtz operator is constructed. This stage presumes massive parallelization using MPI. When LU factorizations have been calculated for a chosen frequency band, iterative eigen-decomposition consists of a number of back-substitutions. Having some predefined integer number q, a partial eigendecomposition $A \sim V_q \Sigma_q V_q^{\,T}$, is obtained where the columns of the matrix V are the first q-eigenvectors (q-first right singular vectors of initial system (1)), and the diagonal entries of the matrix $\Sigma = diag(\lambda_i)$ are q- largest eigenvalues of equation (2) arranged in non-increasing order. The estimated model perturbation $\vec{m}_{EST}$ is calculated as $\vec{m}_{EST} = A_r^{+}(\mathrm{Re}\, L^{*}\vec{d}) = [V\Sigma_r^{-1}V^{T}](\mathrm{Re}\, L^{*}\vec{d})$, $r \le q$ where the diagonal entries of the matrix $\Sigma_r^{-1} = diag(1/\lambda_i)$. The index r determines the condition number of the truncated system. Condition number (cond) serves as a regularization parameter which controls the fundamental trade-off between resolution and stability of the inverse problem (Menke, 1994). The condition number corresponds to given number of the right singular vectors. Less noisy observed data allows to use more singular vectors to construct a solution. The optimal choice of the regularization parameter is a difficult problem in real world (Vogel, 2002). For synthetic cases, comparing different scenarios is much simpler because the true model is known as well as the noise distribution. For each inversion scenario when the noise type and level are known and fixed, we scan the regularization parameter r within some range. The optimal r-value corresponds to the minimum of relative mean-squared error (RMSE) with respect to the true model. Inversion with such optimal regularization provides the best achievable results in a given situation; i.e., the result would have the best trade-off between resolution and stability for a given noise level (Silvestrov *et al.* 2012). We demonstrate by example that analyzing the behavior of RMSE as a function of regularization parameter provides important information about particular inversion scenario.

Left singular vectors which are very useful for analysis of information content and data resolution of inversion scenarios (Menke 1994) may be evaluated using our matrix-free approach as a three stages procedure: **i)** $\vec{q} = L\vec{v}_j$; **ii)** $\vec{q} = \lambda_j^{-1/2}\vec{q}$; **iii)** $\vec{u}_j = [\mathrm{Re}\,\vec{q} \quad \mathrm{Im}\,\vec{q}]^{T}$, where $\vec{u}_j$ is a left singular vector; $\vec{v}_j, \lambda_j$ is j – eigenvector and eigenvalue of the matrix A correspondingly. (Note, here we consider the system (1) as a real valued system, i.e. as consisting of the two blocks: the first one corresponds to a real part of the system (1), ($\mathrm{Re}\, L$), the second one corresponds to the imaginary part, $\mathrm{Im}\, L$: $\tilde{L} = [\mathrm{Re}\, L \quad \mathrm{Im}\, L]^{T}$).

Example

As we are interested in application of nonlinear FWI to land seismic data acquired in areas with complex near surface, we naturally want to understand the effects of the free surface on the inversion. More specifically we try to answer the question using the presented methodology: what is the effect of surface related (SR) multiples on the resolution and stability of the acoustic FWI results?

Two scenarios are compared side by side. The first case is with the free surface present and thus generating multiples as is the case for real land data. The second case is without a free surface (an absorbing boundary condition) which may correspond to real data with multiples removed. All other input parameters remain the same. The difference in the input data for the FWI is the presence or absence of scattered events produced by surface related multiples. Figure 1a shows a synthetic velocity model used for the tests. It contains strong near surface velocity variations. The initial model was obtained from travel-time refraction tomography (Figure 1b). The exact perturbation (difference between true and initial model) in terms of squared slowness is shown in Figure 1c. The acquisition geometry consists of 33 source and 496 receivers located at depth of 5 m with a lateral spacing of 300 m and 20 m respectively. It is well know that FWI for smoothed velocity model reconstruction should utilize the low frequency component of the data to succeed. In our case, the frequency-domain inversion is conducted using four frequencies simultaneously: 2.5, 5, 7.5, 10 Hz. Each frequency has an identical contribution in the composite system (1), i.e. $F(\omega_j)=1$. The size of the target domain for eigen-decomposition is 500 by 80 grid points.

For both scenarios the "ideal observed" data is calculated as a product of the corresponding forward maps and "true" perturbation vector: $Am_{TRUE} = p_{Ideal}$, so "nonlinearity" of the initial model with respect to the "true" one (i.e. too strong deviation to be treated in linearized inversion) plays no role in the experiment. "Ideal" data have been contaminated by uncorrelated white Gaussian noise. The signal-to-noise ratio was equal to 15 dB in both cases.

In Figure 2 we compare the behavior of normalized eigenvalues. As one can see, the eigenvalues in the case without free surface decay much slower than in the case with the free surface (FS). It means that theoretically one may expect much better model resolution for the scenarios without a free surface. Figure 3 shows relative mean-squared-error (RMSE) for the model with and without SR multiples as a function of condition number. One can observe that FWI scenarios without FS provides much better inversion result (lower RMSE) if the regularization parameter is chosen in an optimal way. However this scenario looks less stable with respect to the choice of regularization parameter, i.e. when regularization level is selected away from the optimal value, RMSE rapidly increases and overtakes similar error for inversion with a free surface. This is particularly visible for large condition numbers that correspond to weaker regularization (cond >500). The results of linearized inversion corresponding to cond=140 are shown in Figure 4. As it has been predicted, scenario without a FS results in much better reconstruction of the squared slowness perturbation. When regularization is selected away from optimal value, inversion without a free surface becomes unstable and delivers poor results. In contrast, inversion of free-surface data with a broad range of different condition numbers from 200 to 1000 is predicted to deliver stable but less resolved results (Figure 5).

Conclusions

We present an SVD-based approach to analyse various FWI scenarios both for resolution and stability. Specifically, we demonstrate that presence of surface-related multiples decrease resolution and increase the stability of (low frequencies) inversion procedure in the presence of uncorrelated noise. Similar analysis can be applied to analyze the effects of minimum frequency, maximum offset, acquisition geometry and other parameters important for FWI. These SVD-based conclusions have been validated by full nonlinear FWI experiments.

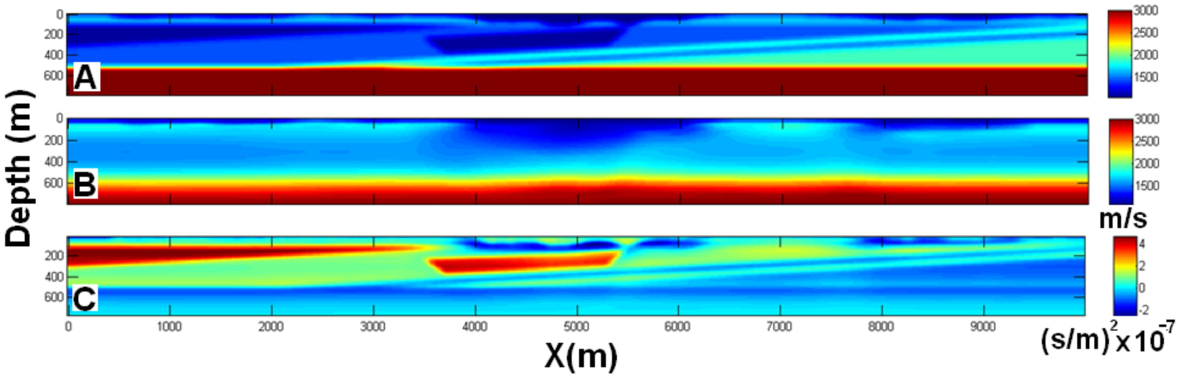

Figure 1 *(A) True Vp model used for SVD analysis; (B) initial Vp model from traveltime refraction tomography; (C) difference between (a) and (b) was used as perturbation in linearized SVD analysis(shown as squared slowness).*

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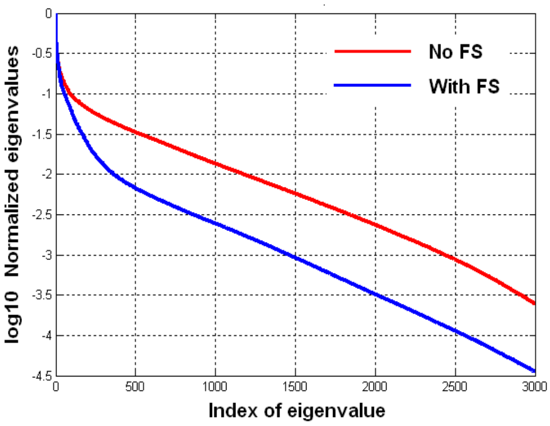

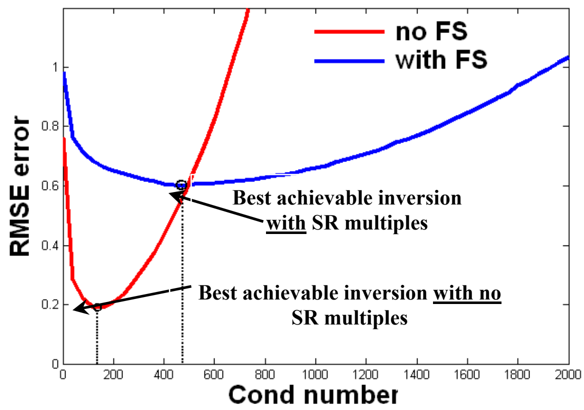

Figure 2 *Normalized eigenvalues for the cases with and without free surface (FS).*
Figure 3 *Relative mean-squared error between exact and estimated models as a function of condition number for cases with and without free surface.*

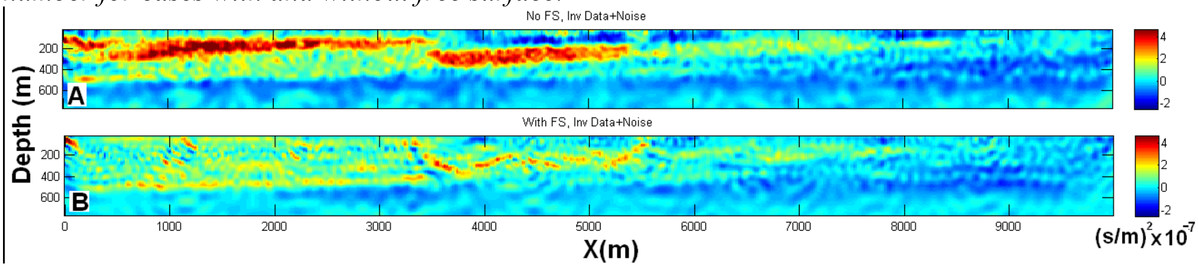

Figure 4 *Truncated SVD inversion for condition number cond=140: (A) without SR multiples; (B) with SR multiples. Chosen condition number corresponds to smallest RMSE error for the case without free surface (Figure 3).*

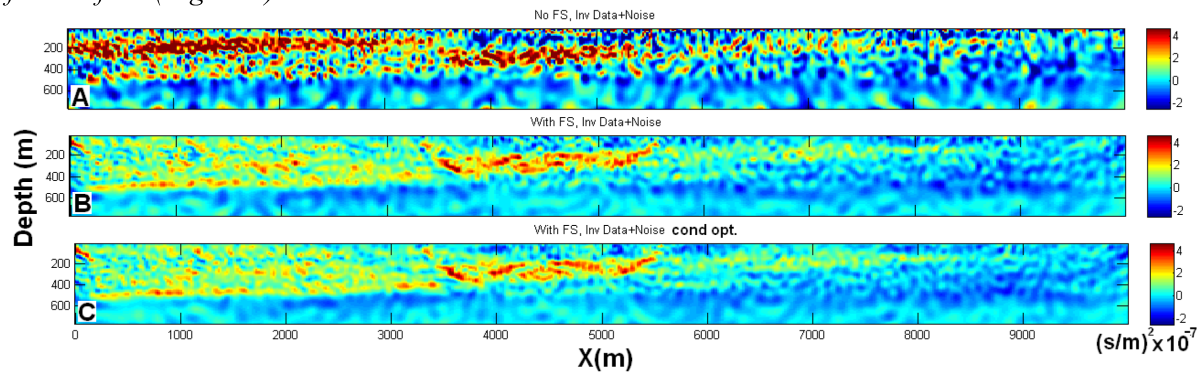

Figure 5 *Truncated SVD inversion for cond=650 for the case without (A) and with SR multiples (B). (C) TSVD inversion for cond=500. This condition number corresponds to smallest RMSE error for the case with free surface (Figure 3). The inversion result with SR multiples is more stable, i.e. contains less artifacts.*

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