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Guided Waves and Rayleigh Leaking Modes With Outpost Algorithm

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SUMMARY

Near surface P-wave structure can be estimated by inverting dispersion curves of the guided waves. Guided waves are associated with special complex roots of dispersion equations. For a model of a solid waveguide overlying a solid half-space, collection of guided waves and Rayleigh leaking modes represent complete set of leaking modes comprising the total elastic wavefield. Dispersion curves inversion requires an efficient, automated and accurate algorithm to compute complex roots of dispersion equation. We propose an application of the so-called outpost method that searches for moments of leaking modes crossing some line in complex plane called an outpost. This approach allows to achieve any desired accuracy by individual continuation of leaking modes as curves parameterized by frequency starting from the outpost. We suggest a particular shape of the outpost line to take into consideration all possible leaking modes arising up to a certain chosen frequency of interest. We test the approach using a near surface model giving rise to strong guided and Rayleigh leaking waves and verify the accuracy of the algorithm by comparison with velocity-frequency spectra obtained by full waveform elastic modeling.

Introduction

Near surface characterization remains crucial challenge for the seismic prospecting community. Strong lateral variation gives rise to the static corrections for the deeper reflection travel times. Whereas seismological methods based on the normal modes dispersion curves inversion by means of Rayleigh waves became a standard procedure (Socco et al., 2010), it suffers from the low sensitivity to the compressional wave velocity. For this reason, the interest to other types of surface waves, especially guided waves predominantly composed by interfering P-waves (Robertsson et al., 1996), is growing (Boiero et al., 2013). Described theoretically by complex roots of dispersion equations or leaking modes, guided waves require special treatment. It was shown that joint inversion of Rayleigh and guided waves is possible by acoustic approximation of dispersion curves in case of a high Poisson ratio of the subsurface waveguide (Roth and Holliger, 1999). Nonetheless, with increase of the shear modulus this approximation fails.

A common technique for the leaking modes search is an extension to the complex plane of the algorithm suggested by Chen (1993) for normal modes, which implies varying of the complex wave velocity for a set of fixed frequency values satisfying the threshold condition (Roth et al., 1998). Despite the efficiency of this approach it has several drawbacks. Due to a fixed constant threshold, it retrieves not the exact values of roots but the domains of roots location. This doesn't lead to an accurate dispersion curves calculation and results in shadows of the varying width (Boiero et al., 2013). Such method is also time consuming because all the grid points of the search space should be checked for a fixed value of frequency. Thereby, there is a demand for a more efficient high-quality algorithm for leaking modes.

Outpost algorithm for leaking modes tracing

An alternative approach for the leaking modes calculation was suggested by Petrashen et al. (1997). It was named an outpost algorithm by the author. The idea of the method has the following theoretical foundation. Almost all the important roots of the dispersion equation with respect to complex slowness come with increasing frequency from the complex infinity of the Riemann surface to the real axis. They concentrate in a vicinity of zero and branch points of complex valued radicals associated with half-space velocities. The other small amount of important roots have their origin at finite points already inside such a domain. Main contribution to the wavefield is determined by the finite number of roots which are close to the real axis. We use the term Riemann surface to emphasize that normal and guided waves leaking modes appear in the complex plane through the branch lines at cut-off frequencies. Before that they move at auxiliary sheets of dispersion equation (Haddon, 1984). A natural approach is to catch the roots at some curve in complex plane (outpost) during their motion to the real axis with increasing frequency. Second step is to continue them individually including crossing of the branch lines. Such a visual procedure equipped with appropriate tools for "catching" and for the "continuing" allows to save the time and to achieve any accuracy of the calculation (Kiselev, 1997).

General dispersion equation has form $\triangle(p,\omega)=0$, where ω is an angular frequency, and p is complex slowness. Inverse real part of p represents phase velocity, whereas imaginary part is responsible for attenuation. Dispersion equation can be parameterized at some curve (outpost) in the complex plane $p=p(s)$, where s is some real parameter, e. g. length of the curve. Variable change yields another equation for positions and moments of the crossing: $\triangle(p(s),\omega)=\widetilde{\triangle}(s,\omega)=0$. This equation in the case of close enough initial approximation can be solved via iterative Newton method:

$$\begin{pmatrix} s_{n+1} \\ \omega_{n+1} \end{pmatrix} = \begin{pmatrix} s_n \\ \omega_n \end{pmatrix} - \alpha_1 \begin{pmatrix} \mathrm{Re}\dfrac{\partial\widetilde{\triangle}}{\partial s} & \mathrm{Re}\dfrac{\partial\widetilde{\triangle}}{\partial\omega} \\ \mathrm{Im}\dfrac{\partial\widetilde{\triangle}}{\partial s} & \mathrm{Im}\dfrac{\partial\widetilde{\triangle}}{\partial\omega} \end{pmatrix}^{-1} \begin{pmatrix} \mathrm{Re}\widetilde{\triangle} \\ \mathrm{Im}\widetilde{\triangle} \end{pmatrix}(s_n,\omega_n). \tag{1}$$

Relaxation parameter $\alpha_1 \le 1$ limits the increments and provides some options to control convergence rate. The initial points sets $\{s_1,s_2,\dots\}$ and $\{\omega_1,\omega_2,\dots\}$ are specified in advance. For time saving purpose, pairs should be followed through the selection. If at some iteration a pair s_i, ω_j leaves rectangle $(s_{i-1}, s_{i+1}) \times (\omega_{j-1}, \omega_{j+1})$, this pair is excluded as not being close to the solution (Kiselev, 1997).

There are several possibilities to continue a leaking mode $p(\omega)$ further from the outpost line. In this study, we employ the Newton method with small steps in ω after convergence in p:

$$p_{n+1} = p_n - \alpha_2 \left(\frac{\partial \triangle}{\partial p} \right)^{-1} \triangle\,(p_n, \omega). \tag{2}$$

The $\alpha_2 \leq 1$ parameter plays the same role as in equation (1). The drawback of this approach is an unpredictable behavior of the closely spaced roots; however, it can be addressed by modification described by Kiselev (1997). The domains of closely spaced roots usually occur in the vicinity of symmetrical leaking mode fusion points (Figure 3) and modes accumulation points.

We supplemented the algorithm by a special choice of the outpost curve providing automation of the procedure. The concept is to surround the vicinity of branch points so that no root can pass close to real axis without crossing the outpost. Leaking modes appear also from behind the branch lines. As a consequence, closed curve on the main $(++)$ sheet with initial integration contour is not enough. Instead we select closed curve on the whole Riemann surface shown as dotted black line marked in Figures 1 and 3. The rectangular configuration is natural for the separating of equally weak modes. Additional tracing of initially embedded roots makes it possible to control all the leaking and normal modes in the domain of interest up to certain maximum frequency determined by source signal.

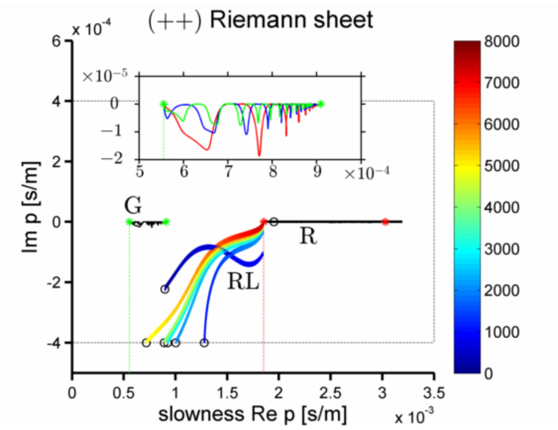

Figure 1 *Guided waves leaking modes, Rayleigh leaking and normal modes on the main sheet.*

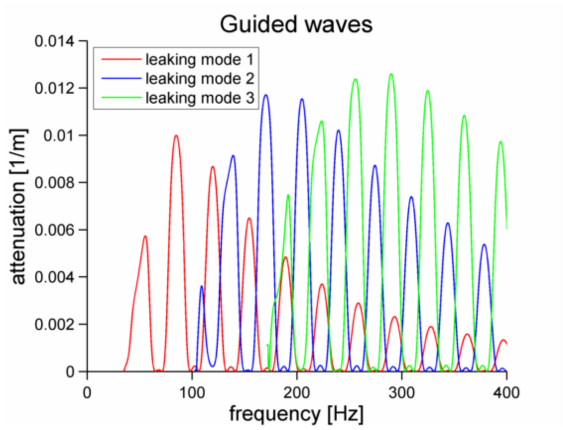

Figure 2 *Attenuation curves of the guided waves leaking modes.*

Guided waves and Rayleigh leaking modes in solid layer over solid half-space

Roth et al. (1998) computed the dispersion and attenuation curves of one guided waves leaky mode for a solid layer over solid half-space model, but the results were not entirely accurate due to imperfection of their algorithm. Surkov and Reshetnikov (2006) applied the outpost strategy to the problem of a solid layer covering a solid half-space and retrieved a variety of Rayleigh leaking modes curves.

We tested our algorithm for a certain set of material parameters. Figures 1, 3 represent the result of leaking modes tracing for a near surface model suggested by Roth and Holliger (1999) with waveguide elastic parameters $v_{p1} = 1.1\ km/s,\ v_{s1} = 0.33\ km/s,\ \rho_1 = 1.6\ g/cm^3$ and underlying bedrock properties $v_{p2} = 1.8\ km/s,\ v_{s2} = 0.54\ km/s,\ \rho_2 = 2\ g/cm^3$. Near surface waveguide has layer thickness $h = 10\ m$. The leaking modes curves are colored with respect to a waveguide parameter $\kappa = \omega h$ (Figure 1, 3), which is more justified to use than just frequency in this case of a single layer over a half-space. Circles indicate initial points of root tracing. Normal modes domains are black line segments. Green asterisks mark P-wave slownesses while red ones denote S-wave slownesses. Here we adopt a convention, that branch lines are chosen going down and have the appropriate color.

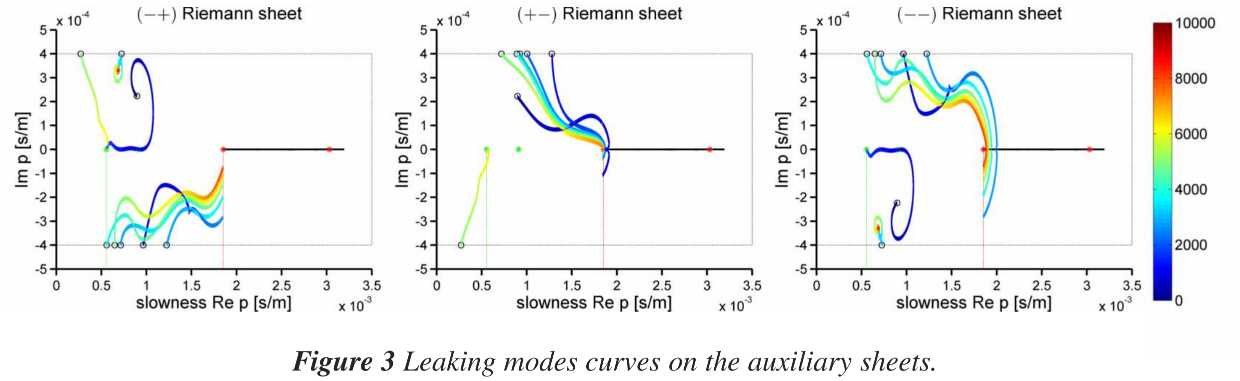

Figure 3 *Leaking modes curves on the auxiliary sheets.*

Every leaking mode curve has a twin on the Riemann surface. This property is related to the dispersion equation symmetries with respect to complex conjugation. Rayleigh leaking modes are reverse continuations of Rayleigh normal modes behind the cut-off frequencies. They originate from the bottom of the outpost line at main $(++)$ sheet (Figure 1) with one exception, which starts from the interior. They have a strong attenuation and only in the vicinity of the crossed branch line come enough close to the real axis. Leaking modes of the guided waves behave similar to acoustic normal modes. They appear from the second $(-+)$ sheet at cut-off frequencies and follow the same path between compressional slownesses, but with some perturbations (Figures 1, 3).

Three consistent guided waves leaking modes are presented at zoomed display in Figure 1. Attenuation curves $2\pi f|\text{Im}p(f)|$ for them are plotted in Figure 2. One can observe the pronounced tuning effect of slowness imaginary part vanishing at certain frequencies (Roth et al., 1998). The amplitude of such oscillations has maxima and diminishes when approaching P-wave slownesses. Oscillation periodicity increases with the leaking mode number and κ.

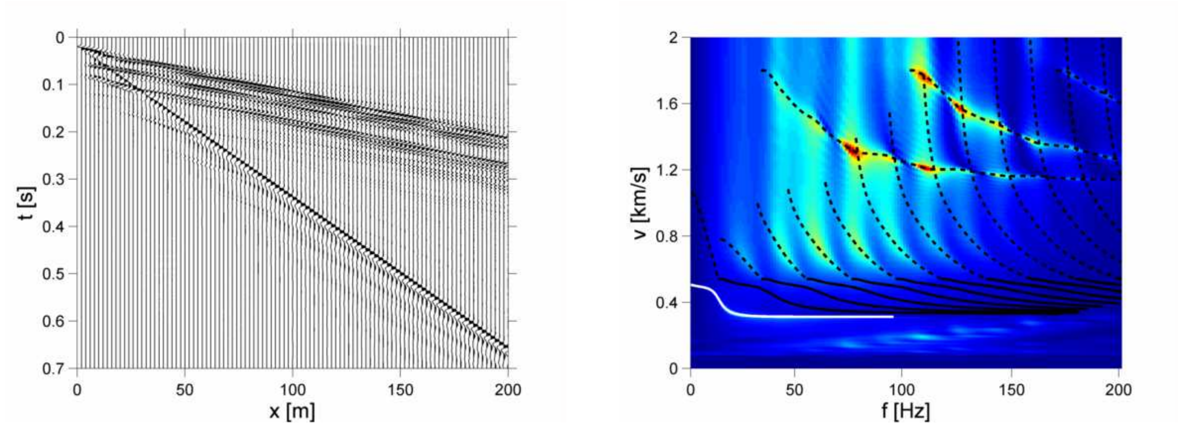

Figure 4 *Shot gather for center of compression source and velocity-frequency $(v-f)$ transform with overlaid theoretical dispersion curves.*

Tests on synthetic data

For the model at hand let us simulate full elastic wavefield from various point sources with Ricker wavelet and central frequency of 100 Hz. Figure 4 shows response for center of compression source at 2 m depth, whereas Figure 5 displays ten times amplified response of a horizontal point force at the same depth. Both synthetic shot gathers are trace normalized. Center of compression strongly excites guided waves, whereas horizontal point force excites Rayleigh leaking waves too. Rayleigh leaking waves have even higher amplitudes. Note that Rayleigh leaking waves appear in the optimal reflection window (Roth and Holliger, 1999) and may cause misidentification of arrivals when reflected events from deeper

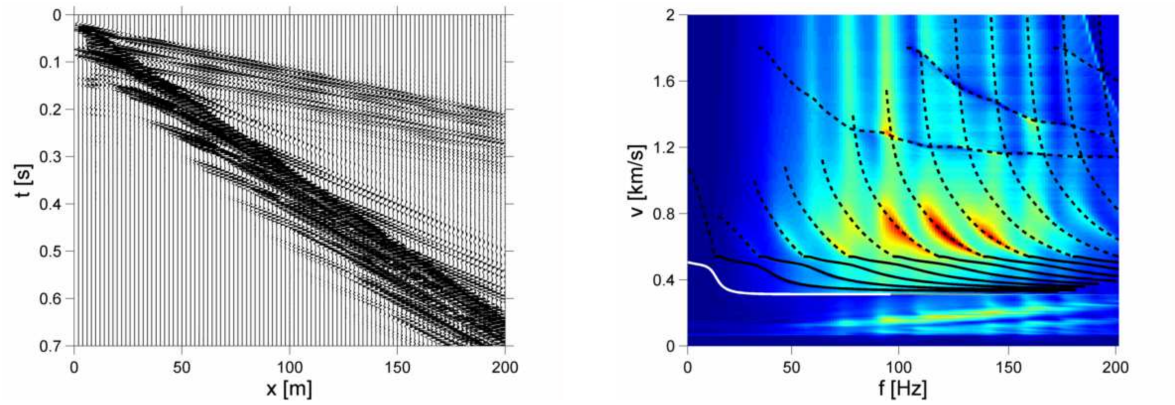

***Figure 5** Shot gather for horizontal force source and corresponding velocity-frequency $(v-f)$ transform with overlaid theoretical dispersion curves.*

horizons are present. Velocity-frequency $(v-f)$ transform was carried out with slant stacks followed by a Fourier transform in time. All the theoretical dispersion curves were obtained with the automatic outpost algorithm. Dashed curves correspond to the leaking modes, whereas solid lines represent the normal modes. Rayleigh fundamental mode is marked by white line.

Conclusions

The new version of outpost algorithm for leaking modes calculation is proposed and tested for a near surface model of solid waveguide over solid half-space. For the first time the leaking modes curves corresponding to the guided waves were fully and precisely calculated. Computed dispersion curves are in complete agreement with velocity-frequency spectra obtained from full-waveform modeling, thus validating efficiency and accuracy of our fully automated algorithm.

Acknowledgements

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