B035

# Wave Field Excitation in Thin Fluid-filled Fracture of Finite Size by External Seismic Wave and its Interaction

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## SUMMARY

The problem on excitation and propagation of slow eigen mode in thin fluid-filled fracture under action of external seismic wave is considered in the report. Based on averaging procedure the non-uniform pseudo-differential wave equation for slow eigen mode in fluid-filled fracture is derived in space-time representation for the long wave approximation by fracture opening. The derived wave equation takes into account strong dispersion of slow eigen mode and contribution of incident external seismic field. The space-time description of slow eigen mode in fracture allows to state and solve the boundary problem on the slow mode excitation by fracture tips and to estimate the contribution of this effect at the tube wave generation in a well. Numerical calculation show that the magnitude of this effect can achieve several percents of the principal tube wave amplitudes and hence it can be used for fracture size characterization.

**Introduction:**

Tube wave generated and propagated in a well contain an important information about properties of surrounding medium and, in particular, about cracks or fractures crossing a well. Knowledge of fracture geometry and its linear sizes has a critical importance for hydraulic fracturing. There are two important cases of the vertical and the horizontal or inclined fractures. In the first case a fracture intersects a well along a finite interval corresponding to its linear size. Hence in this case fracture size can be estimated by time delay between tube wave reflections from top and bottom of fractured zone. In the second case the intersection of a well and fracture is effectively a point. Thus, the interaction of tube wave with a fracture is usually considered in the limit of infinite plane fluid layer [1,2], or, on contrary, as a crack of small wave size [3]. In the both cases the question about fracture size is absent. The attempt to account finite size effect of horizontal fracture at tube wave reflection was mentioned by [2].

However there is additional opportunity to estimate extension of horizontal or inclined fracture by use of tube wave excitation in a well under action of external seismic field. If a fracture crossing a borehole has linear sizes bigger or comparable with wavelength of external seismic wave, than the wave field in fracture fluid can be excited not only in the point of well-fracture crossing but also by fracture tips. This fact was not considered in previous investigations. Due to fluid connection between fracture and a well the both these events generate corresponding tube waves in a well. If these tube waves can be registered in a well, than the linear size of fracture can be estimated by time delay between these tube waves. The key question in this approach is an ability to register tube waves produced by slow eigen mode traveling along fracture from its tips, where it is generated by external seismic wave.

Thus, there is the problem on excitation of the pressure wave field in a well, crossing fluid-filled fracture of finite size, by an external seismic wave field. The results for infinite fracture and crack of small wave sizes have to be the limiting cases of this statement. The statement of the problem is shown in the fig.1.

The smallness of fracture opening $2\delta$ and a well radius $R$ in comparison with seismic wavelength allows us to write the averaged by cross sections acoustical equations for dynamic values in a well and fracture fluids. This approach for derivation of wave equation in a well was successfully applied in [4,5]. In addition, for simplicity, the external seismic field is considered in the local plane wave approximation.

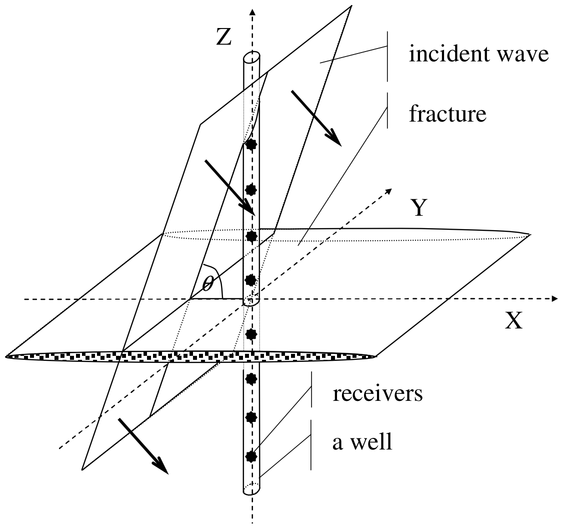


Fig.1. Geometry of the problem

**Wave field propagation in thin fluid-filled fracture of finite size:**

By use of the averaging approach developed in [4,5] it is possible to derive the following equation for pressure in the fracture averaged by its opening:

$$\frac{1}{c_f^2}\frac{\partial^2 P}{\partial t^2} - \Delta_\perp P = -\rho_f \frac{\partial^2}{\partial t^2}\left(\frac{u_z(z=\delta) - u_z(z=-\delta)}{2\delta}\right) \tag{1}$$

where $\rho_f$ and $c_f$ are the fluid density and sound velocity and $u_z(z=\pm\delta)$ are vertical component of the fluid displacements vector in the vicinity of the crack sides under action of external seismic field and fluid pressure $P(x,y,t)$.

The crack disclosing can be found by solution of the dynamic problem about seismic wave reflection from boundary between elastic and fluid half-spaces with the given pressure $P(x, y, t)$ applied to the boundary. In terms of Fourier's transformations by space and time it can be written as:

$$\frac{u_z(+\delta)-u_z(-\delta)}{2\delta}=\frac{\nu_l(\omega/c_s)^2}{D(k,\omega)}\frac{P+\sigma_{zz}^{\Sigma}}{\delta\rho_s c_s^2} \tag{2}$$

where $\omega$, $k_x$, $k_y$ - frequency and wave vector components of plane waves by which the external field can be decomposed, $\rho_s$ - is density of the elastic medium, and $c_l$, $c_s$ - longitudinal and transversal waves velocities, $k^2 = k_x^2 + k_y^2$, $\nu_l^2 = k^2 - \omega^2 / c_l^2$, $\nu_s^2 = k^2 - \omega^2 / c_s^2$. The denominator in (2) corresponds to Rayleigh's dispersion function

$$D(k,\omega) = 4k^2\nu_l\nu_s - \left(k^2+\nu_s^2\right)^2$$

Correspondingly $P(k_x,k_y,\omega)$ is Fourier's transform of the pressure field in fluid and $\sigma_{zz}^{\Sigma}(k_x,k_y,\omega)$ is the total normal stress applied to the both sides of fracture.

Applying the Fourier transforms to equation (1) and substituting the relation (2) we obtain the equation, which in the long wave approximation describes the pressure field $P(k_x,k_y,\omega)$ in the thin fluid layer between two elastic half-spaces under action of external stress field:

$$\left(k^2-\left(\frac{\omega}{c_f}\right)^2-\frac{\rho_f}{\rho_s}\frac{\nu_l(\omega/c_s)^2}{\delta D(k,\omega)}\right)P=\frac{\rho_f}{\rho_s}\frac{\nu_l(\omega/c_s)^2}{\delta D(k,\omega)}\sigma_{zz}^{\Sigma} \tag{3}$$

That defines the dispersion equation for eigen modes in the thin fluid layer between two elastic media as:

$$k^2-\left(\frac{\omega}{c_f}\right)^2-\frac{\rho_f}{\rho_s}\frac{(\omega/c_s)^2(\nu_l/\delta)}{4k^2\nu_l\nu_s-\left(k^2+\nu_s^2\right)^2}=0 \tag{4}$$

This is well known dispersion equation for slow symmetrical mode in thin fluid layer between elastic half spaces [6]. In the low-frequency limit the dispersion of phase velocity for this mode can be described by the following expression:

$$c(\omega)\approx c_s\left(\frac{\omega\delta}{c_s\Delta}\right)^{\frac{1}{3}} \qquad \text{where} \qquad \Delta=\frac{\rho_f/\rho_s}{2\left(1-(c_s/c_l)^2\right)} \tag{5}$$

**Space-time equation for slow eigen mode in fracture:**

As it easy to see the effects of finite size of fracture at this approach are not taken into account because of Fourier transformation technique, which is applicable for infinite plane layered structures only. To describe fractures of finite size we have to derive governing equation in space-time representation. To obtain that result let's slightly modify the equations (3) and (4). In the long wavelength approximation $k\delta << 1$ and $\omega\delta/c_s << 1$ it is possible to make the following asymptotic replacement for dispersion curve of the slow eigen mode:

$$D(k,\omega)\approx \nu_{eff}\nu_l/\Delta \tag{6}$$

where: $\nu_{eff}^2 = k^2 - \omega^2 / c_{eff}^2$ with $c_{eff} = 2c_s \sqrt{\dfrac{1-(c_s/c_l)^2}{3-(c_s/c_l)^2(2-(c_s/c_l)^2)}}$

The accuracy of this asymptotic expansion can be verified by direct comparisons of dispersion curves for the exact equations (4) and for the one with approximation (6), which coincides with acceptable accuracy of few percents.

Taking into account approximation (6) we can rewrite equation (3) in the form:

$$\left(k^2 - \left(\frac{\omega}{c_f}\right)^2 - \frac{\Delta}{\delta}\frac{(\omega/c_s)^2}{\nu_{eff}}\right)P = \frac{\Delta}{\delta}\frac{(\omega/c_s)^2}{\nu_{eff}}\sigma_{zz}^{\Sigma} \qquad (6)$$

The equation (6) in space-time representation is a pseudo-differential wave equation and it can be written as:

$$\frac{1}{c_f^2}\frac{\partial^2 P}{\partial t^2} - \Delta_\perp P + H\left[P - \sigma_{zz}^{\Sigma}\right] = 0, \qquad (7)$$

where for the one-dimensional problem, shown in fig.1 (fracture restricted in the one direction only), operator $H[P]$, has the following representation:

$$H[P] = \frac{\Delta}{c_s^2}\frac{\partial^2}{\partial t^2}\left(c_V \int_0^t d\tau \int_{-L}^{L} \frac{dx'}{\pi\delta} P(x',t) \frac{\theta\left(c_V\tau - |x-x'|\right)}{\sqrt{(c_V\tau)^2 - (x-x')^2}}\right)$$

Analogous result with a few different kernel can be obtained and for general case of two-dimensional fracture with arbitrary perimeter shape.

The derived space-time representation for slow eigen mode in fluid-filled fracture is an analog of wave equation for tube wave in a well[4]. Thus, to describe wave field in a well-fracture system under action of external seismic wave we have two governing equations: 1) for tube wave and 2) for slow fracture mode (7), and to state any problem we have to formulate boundary conditions in the cross point of a well and fracture and on the fracture tips. The first condition corresponds to equality of fluid pressures and mass fluid flows in the cross point. To formulate the boundary condition on the fracture tips, taking into account squeezing effect, we can use the same approach as in [3]:

$$\frac{\partial P}{\partial x} - = \frac{1}{6}\frac{\rho_f L_{tip}}{\rho_0 c_s^2}\frac{\partial^2}{\partial t^2}\left(P + \sigma_{zz}^{\Sigma}\right), \quad x = \pm L, \qquad (8)$$

where: $L_{tip}$ -is effective length of fracture tips.

## Numerical results

As an example, we consider excitation of the wave field in the system of a well and crossing it one-dimensional fracture, which are shown in fig.1. External seismic field is considered as a plane wave $\sigma_{zz}^{\Sigma}(x,t) = f(t - x/c_v)$, $c_v = c_l / \cos\theta$ - is visible velocity of the external seismic wave propagation along the fracture ($\theta = \pi/3$). The fluid parameters correspond to water $\rho_f = 1$ g/cm$^3$, $c_f = 1.5$ km/s and parameters of elastic medium are following $\rho_s = 2$ g/cm$^3$, $c_l = 4.5$ km/s, $c_s = 2.5$ km/s, the chosen linear sizes of fracture have typical values: $L = 50$ m, $\delta = 0.01$ m, $L_{tip} = 5$ m.

Fig.2 represents wave field of tube waves in a well. Fig.2a corresponds to the case of infinite fracture. It is possible to see that the pressure field in a well contains enough rich information as about incident and reflected and transmitted on the fracture body waves, as about intensive tube waves generated at the cross point of the well and fracture. By use of VSP processing technique it allows to extract information about fracture orientation and together with amplitude analysis of tube wave – about fracture opening. Fig.2b represents the difference with the previous picture due to additional tube wave contribution dealt with generation of slow eigen mode only. Again the amplitudes of tube waves related with fracture tips consist of few percents of the principal contributions and hence this effect can be registered. It opens the easy way for independent estimation of fracture sizes by inside borehole acoustical measurements.

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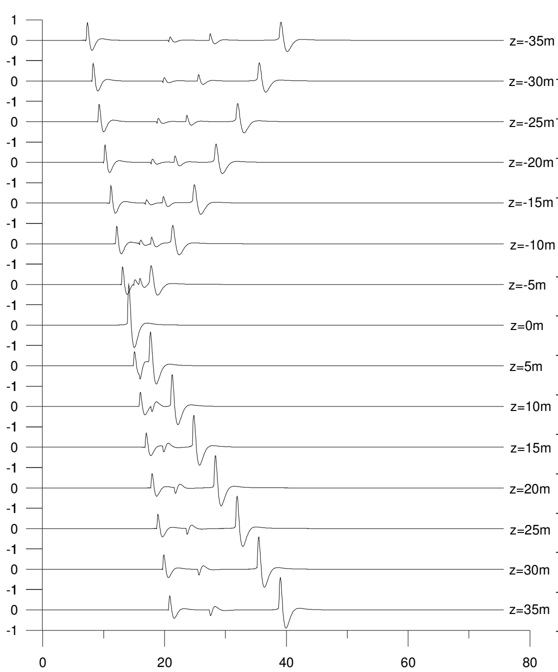


Fig.2a. Seismograms of pressure along a well crossing infinite fracture under action of external seismic wave.

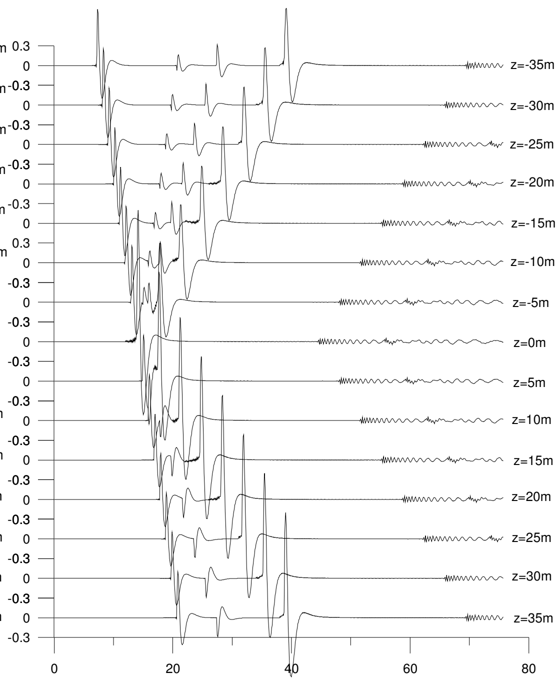


Fig.2b. Seismograms of pressure along a well crossing finite fracture under action of external seismic wave.

## References

1. Tang, X.M., C.H. Cheng, A dynamic model for fluid flow in open borehole fractures // Geophys. Res., 1989, **94** , p. 7567-7576.
2. Hornby, B.E., D.L., Johnson K.W., Winkler R.A. Plumb, Fracture evaluation using reflected Stoneley-wave arrivals // Geophysics, 1989, **54**, p.1274-1288.
3. Derov, A.V., Maximov G.A., Determination of crack orientation in the vicinity of a borehole by vertical seismoacoustic profiling // Acoustical Physics, 2002, **48,** p. 284-291.
4. Ionov A.M., Maximov G.A. Propagation of tube waves generated by an external source in layered permeable rocks. // Geophys. J. Int., 1996, V.124, N 3, p.888-906.
5. Derov, A.V., Maximov G.A., Hydrofracture in external seismic field // XVI Session of Russian Acoustical Society. Moscow, November 14-18, 2005. Moscow, GEOS, 2005.V.1 p.324-327.
6. Ferrazini, V., Aki K., Slow waves trapped in a fluid-filled infinite crack: Implications for volcanic tremor: // J. Geophys. Res. B, 1987, **92**, p.9215-9223.