P023

Tube-Wave Interaction with a Fluid-Filled Circular Fracture of a Finite Radius

S.R. Ziatdinov* (St.-Petersburg University), A.V. Bakulin (Shell International Exploration and Production Inc) & B.M. Kashtan (St.-Petersburg University)

SUMMARY

Tube waves in boreholes are used for characterizing formation properties and hydraulic properties of fluid-filled fractures and permeable zones intersecting wellbores. At low frequencies there is a well-known approximate formalism describing reflection/transmision of tube waves on layer boundaries, infinite fluid-filled fractures and small-diameter washouts. However for fractures or washouts of finite size one can only use numerical methods such as finite difference that are time-consuming and do not provide physical insights.

Here, we extend existing formalism to analyze reflection/transmission of tube waves on a circular fluid-filled fracture/washout of an arbitrary finite size. We break the problem into three tasks. First, conversion of tube waves into guided waves is modeled using method of Kostek et al. (1998). Second, we derive new analytical solution for reflection of diverging guided slow wave from a fracture tip. Finally, we derive conversion coefficient describing transformation of imploding guided into tube waves in a borehole. Combining three solutions, we obtain simple analytical representation of a total wavefield in the borehole as a superposition of upgoing and downgoing tube waves generated at the fracture intersection and borne by incoming guided waves and their multiples. New solution is in good agreement with finite difference computations.

Theory

Let us consider a wellbore of a radius a intersected by a circular fracture with radius b and thickness h (Figure 1a). Propagation of tube waves from a source in a borehole can be decomposed into the following steps: 1) incident tube wave from the source interacts with the fracture resulting in reflected and transmitted tube waves as well as converted guided wave in the fracture (Figure 1b). Conversion coefficient Q_{tg} quantifies amplitude of guided wave excited by unit-amplitude tube wave. 2) Exploding guided wave in the fracture reflects from the fracture tip with reflection coefficient R_{tip} and generates an imploding guided wave approaching the borehole (Figure 1c). 3) When imploding guided wave reaches the borehole it reflects from the fracture mouth with reflection coefficient R_{mth}. The same imploding guided wave impinges onto the borehole in a radially symmetric fashion and generates converted up- and downgoing tube waves in the borehole (Figure 1d).

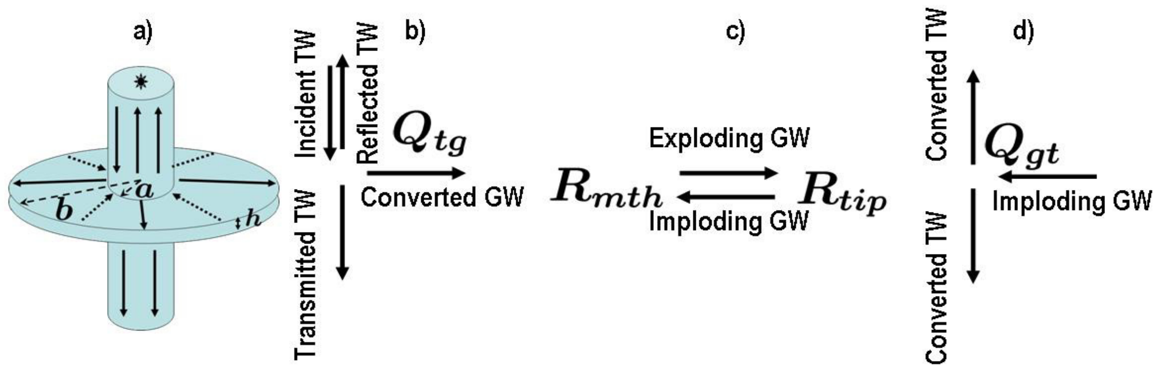

Figure 1. (a) Model geometry. (b)–(d) Simple representation of the wave propagation sequence.

As a result the wavefield inside the fracture can be represented as a following sum of standing (exploding and imploding) guided waves

$$P_{frac}(\omega) = \sum_{n=1}^{\infty} \left(G_e^{(n)} H_0^{(2)}(k_r r) + G_i^{(n)} H_0^{(1)}(k_r r) \right), \qquad (1)$$

$$G_e^{(1)} = Q_{tg}, G_i^{(1)} = G_e^{(1)} R_{tip}, G_e^{(2)} = G_i^{(1)} R_{mth}, G_i^{(2)} = G_e^{(2)} R_{tip}, \ldots, \qquad (2)$$

where $H_0^{(1,2)}$ is Hankel functions, $k_r = \frac{\omega}{V_{GW}}$ is a wavenumber of guided wave, ω is a radial frequency and $V_{GW}(\omega)$ is phase velocity of the slow (symmetric) guided wave in the layer studied by Krauklis (1962) and by Ferraziini and Aki (1987). Temporal dependence $e^{i\omega t}$ is understood but omitted in all equations. Analyzing figure 1 and equation (2), one can see that the first exploding wave ($G_e^{(1)}$) is generated by a tube-to-guided wave transmission with a conversion coefficient Q_{tg}. Amplitude of the first imploding wave ($G_i^{(1)}$) has an additional multiplier R_{tip} and so on and so forth. Multiple guided wave of the order n can be represented as $G_e^{(n)} = Q_{tg} R_{tip}^{(n-1)} R_{mth}^{(n-1)}$ and $G_i^{(n)} = Q_{tg} R_{tip}^{n} R_{mth}^{(n-1)}$.

Likewise pressure wavefield in a borehole above the fracture P_{bor}^{u} and below the fracture P_{bor}^{l} are given by the following sums

$$P_{bor}^{u}(\omega) = e^{-ik_T z} + e^{ik_T z}\left(R + \sum_{n=1}^{\infty} T_c^{(n)} \right), \quad P_{bor}^{l}(\omega) = e^{-ik_T z}\left(T + \sum_{n=1}^{\infty} T_c^{(n)} \right), \qquad (3)$$

$$T^{(1)} = Q_{tg} R_{tip} Q_{gt}, T^{(2)} = Q_{tg} R_{tip} R_{mouth} R_{tip} Q_{gt}, \ \ldots \ , T^{(n)} = G_i^{(n)} Q_{gt}, \qquad (4)$$

where $k_T = \frac{\omega}{V_{TW}}$, $V_{TW}(\omega)$ - phase velocity of the tube wave. Pressure field P_{bor}^{u} consists of a unit-

amplitude downgoing incident tube wave, reflected (upgoing) tube wave from the fracture mouth and infinite number of upgoing converted tube waves generated each time when standing guided wave in the fluid layer hits the fracture mouth. Here conversion coefficient Q_{gt} describes single act of guided-to-tube wave transfer. Likewise, below the fracture, pressure in the borehole consists of transmitted tube wave and infinite number of downgoing converted tube waves. It is straightforward to see that the first member of the series ($T^{(1)}$) is borne by first returning guided wave that reflects once from the fracture tip, while second ($T^{(2)}$) is borne by a guided wave reflected twice from the tip and so on.
Equations (1)–(4) represent total wavefield inside the borehole and fracture as an infinite sum of elementary waves. In practice recording time is finite and therefore only few initial members of the sum need to be computed to predict the response. Also note that each additional reflection reduces the amplitude of subsequent term and therefore contribution of higher-order terms becomes smaller and smaller.
Conversion and reflection coefficients needed to apply equations (1)–(3) can be established in simple analytical form provided borehole diameter and fracture thickness are small compared to wavelength of tube or guided waves. Kostek et al. (1998) provided analytic solution for tube-wave reflection and transmission coefficients $R(\omega)$ and $T(\omega)$ as well as for conversion coefficient $Q_{tg}(\omega)$ using a model of an infinite fracture. These expressions are simple functions of wavenumber of tube waves k_T, wavenumber k_r of guided wave, fracture thickness h and borehole radius a (Figure 1a). In this study, we derive expressions for reflection from the tip R_{tip} and mouth R_{mth} and conversion coefficient Q_{gt} and compare approximate solutions (1)-(4) with finite-difference computations.

Reflection of a cylindrically exploding guided wave from the tip of a circular fracture

Using the fact that the fracture is very thin relative to the wavelength of the slow guided wave, we approximate wave propagation by 2D plane model of fluid-filled circular disk surrounded by elastic medium (Figure 2a). We assume continuity of radial stress component $[t_{rr}]=0$ and displacement $[u_r]=0$ at the fracture tip ($r=b$) and keep phase velocity $V_{GW}(\omega)$ of an original problem with thin fluid layer. Under these assumptions pressure field inside fluid disk from a point source at the center can be found as superposition of Hankel function (direct wave from the source) and Bessel function (standing waves inside the disk). Standing-wave contribution can be expanded as an infinite series that physically describes reverberation of exploding and imploding cylindrical waves. Exploding wave from the source reflects off the fluid-solid boundary, focuses back at the source, then diverges again and repeat this cycle infinite number of times. First term of the series represents desired reflection R_{tip} describing single act of reflection of a guided wave from a fracture tip

$$R_{tip}(\omega)=[\frac{H_0^{(2)}(\frac{\omega}{V_{GW}}b)(1-Za_2)}{H_0^{(1)}(\frac{\omega}{V_{GW}}b)(1-Za_1)}],\ Z=-\frac{\rho_s V_p}{\rho_{fl}V_{GW}}\frac{H_0^{(2)}(\frac{\omega}{V_p}b)}{H_1^{(2)}(\frac{\omega}{V_p}b)}+\frac{2\rho_s V_s^2}{b\rho_{fl}V_{GW}\omega}, \quad (5)$$

where $a_r=\frac{H_1^{(r)}(\frac{\omega}{V_{GW}}b)}{H_0^{(r)}(\frac{\omega}{V_{GW}}b}$, $r=1,2$; V_p, V_s and ρ_s - formation compressional and shear velocity and density, ρ_{fl} - density of the fluid.

Conversion of guided wave into tube wave

Conversion coefficient Q_{gt} describes amplitude of tube wave generated by an imploding cylindrical guided wave of a unit amplitude that symmetrically impinges on a borehole (Figure 1d). Krauklis and Krauklis (1995) solved similar conversion problem without radial symmetry and took into account also diffusive wave generated at an intersection between the layer and the borehole and tube wave generated by elastic vibrations of the wall of the borehole. Here we neglect all the waves except guided wave in the fracture and tube wave in the borehole and hence employ approach similar to Kostek et al. (1998).

We assume that pressure field in the layer consists of only incident and reflected slow guided waves:

$$P_{frac}(\omega) = H_0^{(1)}(k_r r) + R_{mth}(\omega) H_0^{(2)}(k_r r), \qquad (6)$$

while pressure in the borehole consists of two tube waves propagating in the opposite directions:

$$P_{bor}^{l}(\omega) = Q_{gt}(\omega) e^{-ik_T z}, z > 0, \;\; P_{bor}^{u}(\omega) = Q_{gt}(\omega) e^{ik_T z}, z < 0. \qquad (7)$$

Note that up- and downgoing converted tube waves have identical amplitudes because guided mode represents symmetrical mode with respect to the middle plane of the fluid layer.
Analogous to Kostek et al. (1998) we impose boundary conditions requiring continuity of the pressure and continuity of the fluid mass exchanged between the borehole and the fracture. This allows us to obtain simple expressions for reflection and conversion coefficients

$$R_{mth}(\omega) = -\frac{k_T - \frac{ihk_r}{a}\frac{H_1^{(1)}(k_r a)}{H_0^{(1)}(k_r a)}}{k_T - \frac{ihk_r}{a}\frac{H_1^{(2)}(k_r a)}{H_0^{(2)}(k_r a)}} \frac{H_0^{(1)}(k_r a)}{H_0^{(2)}(k_r a)}, \;\; Q_{gt}(\omega) = \frac{\frac{ihk_r}{a}\left[\frac{H_1^{(1)}(k_r a)}{H_0^{(1)}(k_r a)} - \frac{H_1^{(2)}(k_r a)}{H_0^{(2)}(k_r a)}\right]}{k_T - \frac{ihk_r}{a}\frac{H_1^{(2)}(k_r a)}{H_0^{(2)}(k_r a)}} H_0^{(1)}(k_r a). \qquad (8)$$

Comparison of analytical solution with finite difference

Let us compare analytical solution with a finite-difference computation. The point pressure source is located in the borehole above the fracture (Figure 1a). Source has Ricker wavelet with a central frequency of 1000 Hz. Pressure wavefield is computed in the borehole fluid and inside the fracture. We take borehole radius $a = 0.1\,m$ and fracture thickness also $h = 0.1\,m$. Both wellbore and the fracture are filled with water ($V_{fl} = 1500\,m/s$, $\rho_{fl} = 1000\,kg/m^3$), while formation properties are as follows: $V_p = 4200 m/s$, $V_s = 2500\,m/s$, $\rho_s = 2400\,kg/m^3$.

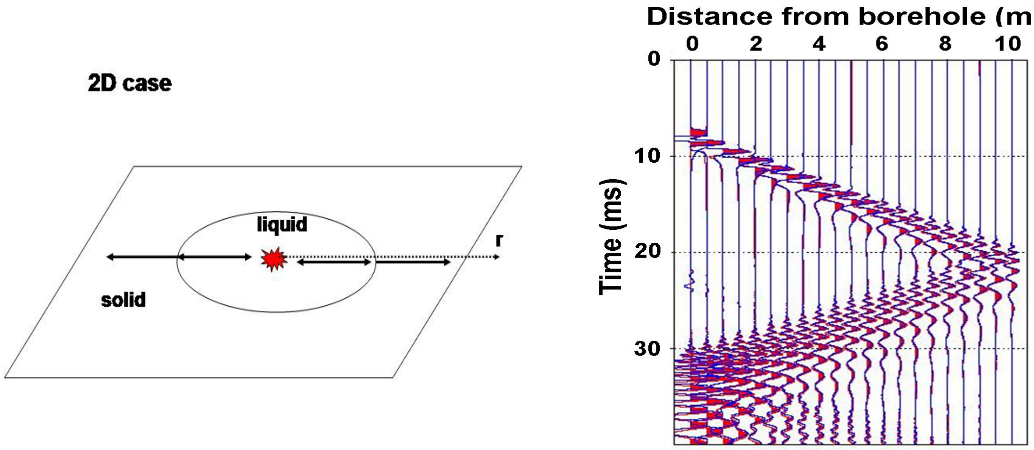

Figure 2. (a) 2D model used for deriving reflection coefficient from the tip. (b) Wavefield inside the fracture with length 10 m.

Figure 2b illustrates wavefield inside the fracture ($b = 10\,m$). Exploding wave originating at □ 10 ms represents guided mode converted from incident tube wave. Red response is computed analytically using equation (1)–(2) and conversion coefficient from Kostek et al (1998) while blue response is finite-difference numerics. Guided wave reflects off the fracture tip at 10 m and generates even more dispersive imploding guided wavepacket that converges towards the borehole center. Finite-difference (blue) and analytical (red) responses are in good agreement for all arrivals thus validating formula (5) for reflection coefficient from the fracture tip.
Figure 3a shows wavefield in the borehole intersecting fracture with 0.36 m radius located at depth of 5 m. Note that waveforms of reflected and transmitted tube-wave packets are more complex and of longer duration compared to incident waveform because they represent interference of several tube waves with short delay time. Analytical response (red) was computed using only first three terms of infinite series (3).

Analytic (red) and finite-difference response (blue) are in good agreement despite this truncation.

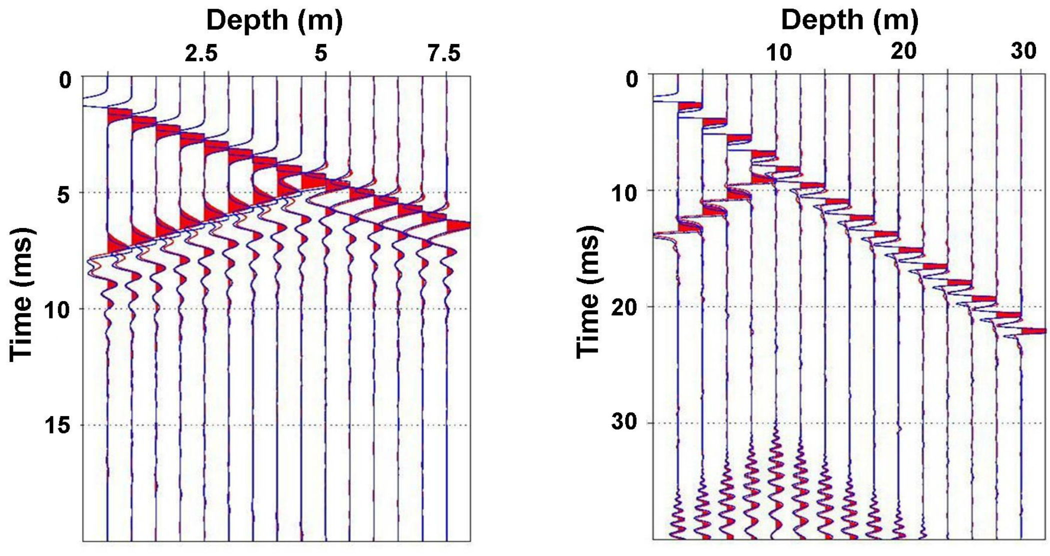

Figure 3. Wavefield in a borehole intersected by a fracture: (a) fracture 0.36 m; (b) fracture 10 m.

For fracture with 10 m radius, multiple reflections from the tip separate in time and so do the tube waves in the borehole. Figure 3b shows that converted tube waves at about □ 30 ms are well predicted by first equation of (4) since analytical (red) and finite-difference (blue) traces closely track each other.

Discussion and conclusions

We have developed a solution to describe interaction of low-frequency tube waves in a borehole with a circular fracture/washout of arbitrary finite radius. Wavefield in the fracture is represented as a sum of successive reverberations consisting of imploding and exploding cylindrical guided waves bouncing between fracture tip and mouth. Imploding guided waves give rise to a corresponding set of converted tube waves in a borehole. We derive formulae for reflection coefficients of guided wave from fracture tip and mouth as well as guided-to-tube wave conversion coefficient. Comparison with finite-difference numerical computation demonstrate that new solution with as few as three reverberations can provide accurate representation of the total wavefield. Since new solution is analytical, we can obtain better physical insight into tube-wave interaction with fractures/washouts of finite radius and analyze dependence of the response on various fracture, borehole and formation parameters.

References

[1] Ferrazzini, V., Aki, K., [1987]. Slow waves trapped in a fluid-filled infinite crack: implication for volcanic tremor. J. Geophys. Res., 92, 9215-9223.
[2] Kostek, S., Johnson, D. L., Winkler, K.W., and Hornby, B.E [1998]. The interaction of tube-waves with borehole fractures, Part II: Analytical models. Geophysics 63, 809–815.
[3] Krauklis, P.V. [1962] On some low-frequency vibrations of a liquid layer in an elastic medium. Priklad. Mat. Mech. 26 (6), 1111-1115 (in Russian).
[4] Krauklis, P.V. and Krauklis, L.A.[1995] Excitation of a tube waves in a borehole by slow wave in a liquid layer. Proc. St.Petersburg Branch of Steklov Math. Inst., in: Math. Problems of Wave Propagation Theory 230 (25), 115-124 (in Russian).