Characterizing hydraulic fractures using slow waves in the fracture and tube waves in the borehole

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Summary

We focus on estimating size of a hydraulic fracture using analysis of tube waves generated by the action of an external seismic field. External seismic field excites not only tube waves at the intersection of the fracture with the well, but also generates slow eigenmode by squeezing the fracture tips. Slow mode propagates along the fracture and converts into tube wave upon reaching the wellbore. Time delay between primary and secondary tube waves allows to estimate fracture size. This study concentrates on describing the secondary tube wave. Using averaging procedure we present the non-uniform pseudo-differential wave equation for slow eigenmode in the fluid-filled fracture. This equation is derived in space-time representation assuming long-wavelength approximation. Derived wave equation takes into account strong dispersion of slow wave and contribution of incident external seismic field. We state and solve the boundary problem describing slow mode excited at the fracture tips and estimate the amplitude of the secondary tube wave generated when slow mode hits the intersection with the well. Numerical calculations show that amplitudes of the secondary tube wave can reach several percents of the primary tube wave amplitudes and hence be detectable in a field experiments and used for fracture size characterization.

Introduction

Propagating tube waves carry an important information about properties of surrounding medium and, in particular, about cracks or fractures intersecting the well (Beydoun et al., 1985; Tang and Cheng, 1989; Hornby et al., 1989; Kostek et al., 1998; Henry et al., 2002, Derov and Maximov, 2002; Ionov, 2007). Knowledge of fracture geometry and dimensions has a critical importance for hydraulic fracturing process. Assuming that the treatment well is vertical, there are two important fracture geometries to consider. In the first case vertical fracture intersects a well along it's entire length. Thus, in this case fracture size can be estimated by time delay between tube wave reflections from the top and bottom of the fractured zone (Medlin and Schmitt, 1994; Paige et al., 1995; Patzek and De, 2000). In the second case of horizontal or inclined fracture the intersection of a well and fracture is effectively a point. Thus, the interaction of tube wave with a fracture is usually considered in the limit of infinite plane fluid layer (Beydoun et al., 1985; Tang and Cheng, 1989; Hornby et al., 1989, Kostek et al., 1998; Ionov, 2007) or, in the contrary, as a crack of small size (Derov and Maximov, 2002). In case of horizontal or inclined fracture determining fracture length is a challenge. The attempt to account for a finite size of the horizontal fracture on tube-wave reflection was mentioned by Hornby et al. (1989) and its experimental verification was undertaken by Henry et al. (2002). Alternative approaches for estimating size of the horizontal or inclined fracture utilized diffraction of external seismic waves on the crack tips (Groenenboom and Falk, 2000; Groenenboom and van Dam, 2000).

However there exist another way to estimate length of horizontal or inclined fracture by using tube waves excited in a well under action of external seismic field. If a fracture crossing a borehole has linear dimensions larger or comparable to the wavelength of external seismic wave, than wavefield in the fracture fluid can be excited not only at the point of well-fracture intersection but also at the fracture tips. This fact was not considered in previous studies. Primary tube wave is excited when external seismic wave hits the intersection between the fracture and wellbore. Slow wave in the fracture is generated when external seismic wave squeezes the fracture tip. This slow mode propagates along the fracture and, upon reaching the wellbore, it converts into another tube wave that we call a secondary. If both of these tube waves can be registered in a well, then the length of the fracture can be estimated from a time delay between their arrivals. The fundamental question is to predict expected amplitudes of these secondary tube waves produced by slow eigenmode traveling along fracture from its tips, when eigenmode is generated by external seismic wave.

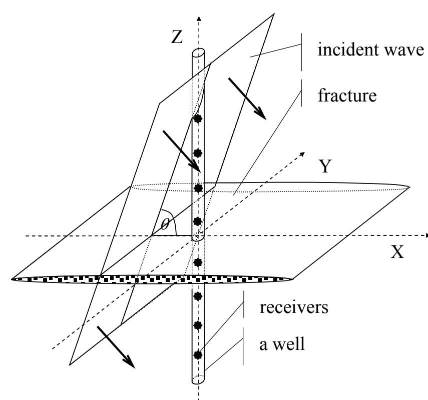

Figure1: Geometry of the problem showing vertical well and inclined hydraulic fracture illuminated by an external seismic wave.

In this study we consider an external seismic wave field that illuminates the well and the fracture. We pose a problem of excitation of a pressure wavefield in a well intersected by arbitrary oriented fluid-filled fracture of a finite size, and generated by an external seismic wave field. The results for infinite fracture and crack of small wave sizes have to be the limiting cases of this statement. The statement of the problem is shown in the Figure 1.

The smallness of fracture opening 2δ and well radius R in comparison with seismic wavelength allows us to write the averaged (by cross section) acoustical equations for dynamic pressure field in a well and fracture fluids. This approach for derivation of wave equation in a well was successfully applied by Ionov and Maximov (1996). In addition, for simplicity, the external seismic field is approximated by a local plane wave.

Wave propagation in thin fluid-filled fracture of finite size

If we define the average pressure in the fracture as $P(x,y,t)=\frac{1}{2\delta}\int\limits_{-\delta}^{\delta} dz P(x,y,z,t)$, then using approach developed by Ionov and Maximov (1996) it is possible to derive the following equation

$$\frac{1}{c_f^2}\frac{\partial^2 P}{\partial t^2}-\Delta_{\perp}P=-\rho_f\frac{\partial^2}{\partial t^2}\left(\frac{u_z(z=\delta)-u_z(z=-\delta)}{2\delta}\right), \quad (1)$$

where ρ_f and c_f are the fluid density and sound velocity and $u_z(z=\pm\delta)$ are vertical components of the fluid displacements vector in the vicinity of the crack sides under action of external seismic field and fluid pressure $P(x,y,t)$.

The crack opening can be found by solving dynamic problem of seismic wave reflection from a boundary between elastic and fluid half-spaces with the given pressure $P(x,y,t)$ applied to the boundary. After Fourier transform in space and time it can be written as

$$\frac{u_z(+\delta)-u_z(-\delta)}{2\delta}=\frac{\nu_l(\omega/c_s)^2}{D(k,\omega)}\frac{P+\sigma_{zz}^{\Sigma}}{\delta\rho_s c_s^2}, \quad (2)$$

where ω, k_x, k_y are frequency and wave vector components of external incident plane wave, ρ_s is density of the elastic medium, and c_l, c_s are speeds of longitudinal and transverse waves, $k^2=k_x^2+k_y^2$, $\nu_l^2=k^2-\omega^2/c_l^2$, $\nu_s^2=k^2-\omega^2/c_s^2$. The denominator in (2) corresponds to Rayleigh's dispersion function

$$D(k,\omega)=4k^2\nu_l\nu_s-\left(k^2+\nu_s^2\right)^2 .$$

Likewise, $P(k_x,k_y,\omega)$ is the Fourier transform of the pressure field in the fluid and $\sigma_{zz}^{\Sigma}(k_x,k_y,\omega)$ is the total normal stress applied to the both sides of the fracture.

Applying the Fourier transforms to equation (1) and substituting the relation (2) we obtain the equation, that, in the long-wave approximation, describes the pressure field $P(k_x,k_y,\omega)$ in thin fluid layer between two elastic half-spaces under action of external stress field

$$\left(k^2-\left(\frac{\omega}{c_f}\right)^2-\frac{\rho_f}{\rho_s}\frac{\nu_l(\omega/c_s)^2}{\delta D(k,\omega)}\right)P=\frac{\rho_f}{\rho_s}\frac{\nu_l(\omega/c_s)^2}{\delta D(k,\omega)}\sigma_{zz}^{\Sigma}. \quad (3)$$

This equation defines the dispersion relation for eigenmodes in the thin fluid layer between two elastic media as

$$k^2-\left(\frac{\omega}{c_f}\right)^2-\frac{\rho_f}{\rho_s}\frac{(\omega/c_s)^2(\nu_l/\delta)}{4k^2\nu_l\nu_s-\left(k^2+\nu_s^2\right)^2}=0. \quad (4)$$

This is a well-known dispersion equation for slow symmetrical mode in the thin fluid layer between elastic half spaces (Ferrazini and Aki, 1987). In the low-frequency limit the dispersion of phase velocity for this mode can be approximated by the following expression

$$c(\omega)\approx c_s\left(\frac{\omega\delta}{c_s\Delta}\right)^{\frac{1}{3}} \quad \text{where} \quad \Delta=\frac{\rho_f/\rho_s}{2\left(1-(c_s/c_l)^2\right)}. \quad (5)$$

Space-time equation for slow eigenmode in a fracture

It is easy to see that effect of finite size of the fracture is not accounted in the above approach because of Fourier transformations, which are applicable only for infinitely long plane-layered structures. To describe fracture of a finite size we have to derive governing equation in space-time representation. To obtain such result let us slightly modify the equations (3) and (4). In the long wavelength approximation, when $k\delta<<1$ and $\omega\delta/c_s<<1$, it is possible to make the following asymptotic replacement for the dispersion curve of the slow eigenmode

$$D(k,\omega)\approx\nu_{eff}\nu_l/\Delta, \quad (6)$$

where $\nu_{eff}^2=k^2-\omega^2/c_{eff}^2$

with $$c_{eff}=2c_s\sqrt{\frac{1-(c_s/c_l)^2}{3-(c_s/c_l)^2(2-(c_s/c_l)^2)}}.$$

Figure 2 verifies the acceptable accuracy of this asymptotic expansion by comparing dispersion curves computed with exact (4) and approximate (6) equations.

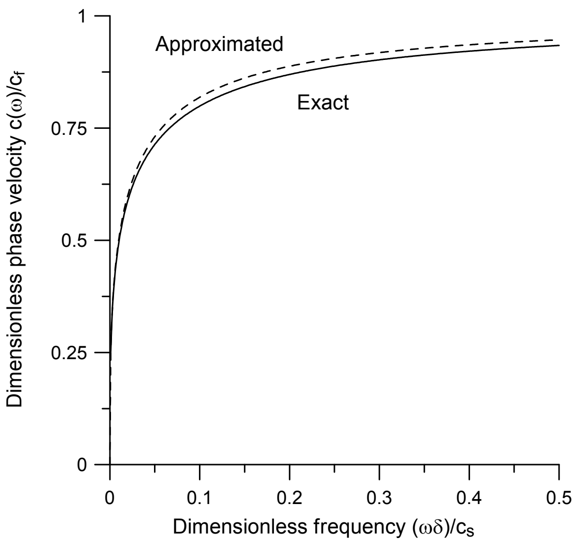

Figure 2: Frequency dependence of phase velocity for the slow eigenmode in thin fluid layer. Solid line – exact dispersion equation (4), dashed line – approximation (6) .

Taking into account approximation (6) we can rewrite equation (3) in the form

$$\left(k^2-\left(\frac{\omega}{c_f}\right)^2-\frac{\Delta}{\delta}\frac{(\omega/c_s)^2}{v_{eff}}\right)P=\frac{\Delta}{\delta}\frac{(\omega/c_s)^2}{v_{eff}}\sigma_{zz}^{\Sigma}. \qquad (6)$$

Equation (6) in space-time representation is a pseudo-differential wave equation and it can be written as

$$\frac{1}{c_f^2}\frac{\partial^2 P}{\partial t^2}-\Delta_{\perp}P+H\left[P-\sigma_{zz}^{\Sigma}\right]=0\,. \qquad (7)$$

For one-dimensional problem, shown in Figure 1 (fracture restricted in one direction and infinite in the other), operator $H[P]$, has the following representation

$$H[P]=\frac{\Delta}{c_s^2}\frac{\partial^2}{\partial t^2}\left(c_V\int_0^t d\tau\int_{-L}^{L}\frac{dx'}{\pi\delta}P(x',t)\frac{\theta\left(c_V\tau-|x-x'|\right)}{\sqrt{(c_V\tau)^2-(x-x')^2}}\right)$$

Similar result with a slightly different kernel can be obtained for the general case of two-dimensional fracture with arbitrary perimeter shape.

The derived space-time representation for slow eigenmode in the fluid-filled fracture is analogous to the wave equation for the tube wave in a well (Ionov and Maximov, 1996). Thus, to describe wavefield in a borehole-fracture system under action of external seismic wave we have two governing equations: one for the tube wave and another for the slow fracture mode (equation 7). In addition, we have to formulate boundary conditions at the intersection between the well and the fracture as well as boundary conditions at the fracture tips. The first condition can be expressed as equality of fluid pressures and mass fluid flows across the intersection. Boundary condition at the fracture tips are derived using approach of Maximov and Ionov (1998) and can be written as

$$P+\frac{\rho_0}{\rho_f}\frac{c_l}{i\omega}\frac{\partial P}{\partial x}=-\sigma_{xx}^0-i\omega\rho_0 c_l u_x^0\,. \qquad (8)$$

Numerical results

Main effect, which has to be verified in the described approach, is the correct description of amplitude of slow fracture mode generated by an external seismic wave on the fracture tips. To perform this verification we used finite-difference modeling code for cylindrically layered media provided to us by Shell Int. E&P. To carry out an exact comparison we reformulated equations (7) - (8) for a cylindrical geometry (see Figure 3).

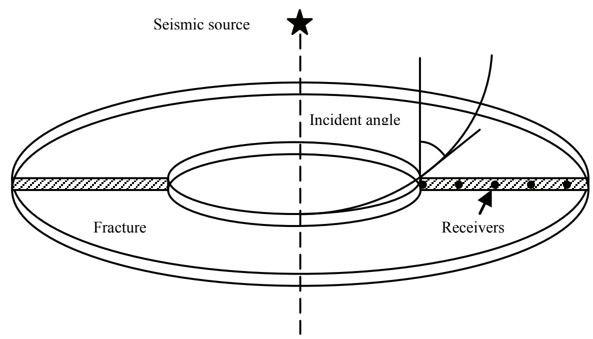

Figure 3: Cylindrical fracture geometry used for verification with a finite-difference modeling. Fracture has a doughnut-shape horizontal cross-section.

Fracture is represented as a thin (1 cm thickness) doughnut-shaped water-filled layer with the inner radius of 4 m. The fracture tips are rectangular. Infill fluid is a water with parameters $\rho_f=1$ g/cm^3, $c_f=1.5$ km/s, whereas elastic host medium parameters are as follows: $\rho_s=2$ g/cm^3, $c_l=4.5$ km/s, $c_s=2.5$ km/s. Point pressure source is located on the symmetry axis. By varying an offset from the source to the fracture plane we can illuminate fracture tips at a different angles. The source radiates spherical pressure pulse with the shape of the second derivative of a Gaussian with a dominant frequency of 700 Hz. In the background elastic medium this creates a disturbance with the longitudinal wavelength of about 6 m. Pressure receivers are placed inside the fracture with a constant spacing of 0.25 m.
Figure 4 represents a comparison of wavefields generated by the developed approach (red line) and by the finite-

difference calculations (black line) when incident angle of the external wave was 45^0 (Figure 3). We observe excellent agreement between the two sets of waveforms including later and weaker arrival of the slow wave generated at the tip. This comparison verifies accuracy of our approach.

Now we are ready to estimate amplitudes of the secondary tube waves in a well excited by slow eigenmode from the fracture tips. As an example, we consider excitation of the wave field in case of a vertical well intersected by one-dimensional fracture infinite in one direction (Figure 1). External seismic field is considered as a plane wave $\sigma_{zz}^{\Sigma}(x,t) = f(t - x/c_v)$, where $c_v = c_l / \cos\theta$ is apparent velocity of the external seismic wave propagation along the fracture. We assume that fracture is characterized by tilt $\theta = \pi/3$, length $L = 50$ m and width $\delta = 0.01$ m. Figure 5 shows seismograms of the total pressure field inside the well. First, notice strong reflected and transmitted tube waves generated when incident P-wave hits the intersection of the fracture with the well. Late weak arrival corresponds to the secondary tube wave that is generated when slow eigenmode from the fracture tips hits the wellbore. Amplitudes of this secondary tube wave is

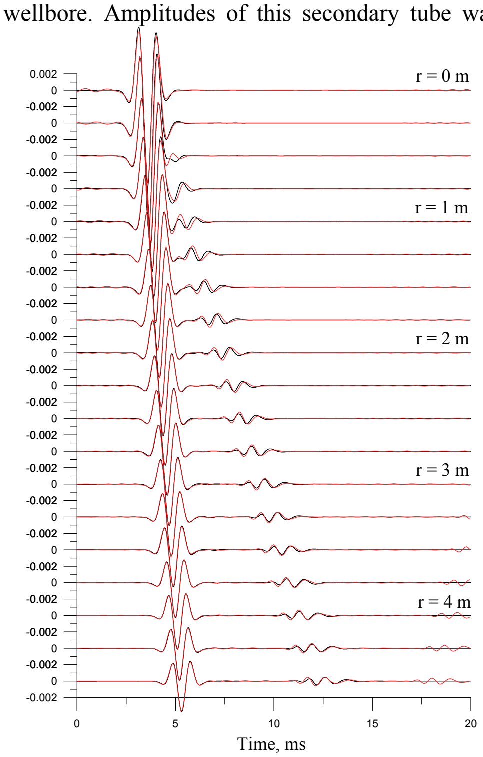

Figure 4: Comparison of pressure seismograms along the fracture when external plane wave illuminates the fracture at 45^0. Black line corresponds to the finite-difference calculation, whereas red line denotes results of the developed analytical approach.

typically several percents (up to ten) of the primary tube wave magnitude. Therefore such arrivals may be potentially recorded in a real field data.

Conclusions

We present an approach that uses tube-wave information from a VSP measurements to estimate fracture size by registering time delays between the primary and secondary tube waves. These secondary tube waves originate when slow wave propagating in the fluid-filled fracture hits the intersection between the fracture and the borehole. We presented analytical approach that correctly computes amplitude and waveforms of the slow fracture mode excited an the external seismic wavefield from an offset source. We validated new approach by comparison with the finite-difference modeling and estimated magnitude of the expected secondary tube waves for realistic fracture-wellbore configurations.

Acknowledgements

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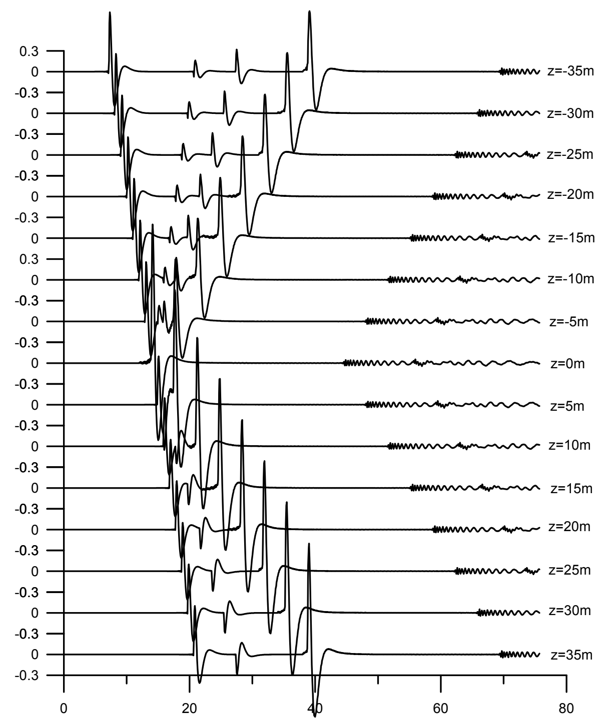

Figure 5: Pressure seismograms along a well intersected by a finite fracture (Figure 1). Wavefield is excited by an external seismic wave. Note strong primary tube waves originated at z=0m. Also observe weak later arrival (>50ms) representing secondary tube wave converted from slow fracture wave.

EDITED REFERENCES

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