P156

ATTENUATION IN THE EFFECTIVE MODEL OF FINELY LAYERED POROUS BIOT MEDIA

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We address the problem of anisotropic attenuation in thinly-stratified poroelastic medium. Effective (long-wave equivalent) model of finely layered poroelastic medium without loss was constructed by matrix averaging method [1]. This model was found to be a generalized transversely isotropic Biot medium with anisotropic densities. Here we extend the model [1] to the case when layers possess viscosity (Biot mechanism) and/or relaxation. This extension does not change the Hooke's law and affects only equilibrium equations which are written in form

$$\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xz}}{\partial z}=\rho\ddot{u}_x+\rho_f\ddot{w}_x,\ \frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{zz}}{\partial z}=\rho\ddot{u}_z+\rho_f\ddot{w}_z,$$
$$-\frac{\partial p}{\partial x}=\rho_f\ddot{u}_x+\mu\ddot{w}_x,\qquad -\frac{\partial p}{\partial z}=\rho_f\ddot{u}_z+\mu\ddot{w}_z, \tag{1}$$

where u_x, u_z - displacements in solid phase, w_x, w_z - relative fluid displacement, τ_{xx}, τ_{zz}, τ_{xz} – total bulk stresses in porous medium, p - fluid pressure, ρ - volume averaged density, ρ_f - fluid density. If we take into account only viscosity (Biot mechanism) then μ operator is given by

$$\mu=m+b/s,\quad m=\rho_f\alpha/\varepsilon,\quad b=\eta/\kappa, \tag{2}$$

where α – tortuosity of pore space, ε – porosity, η - fluid viscosity, κ - permeability and s is a differentiation operator ($s=\partial/\partial t$). Each porous layer in period has a number i ($i=1,2$) and portion θ_i ($\theta_1+\theta_2=1$). Parameters belonging to each layer are marked by subscript i.

After averaging equilibrium equations have a form

$$\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xz}}{\partial z}=\tilde{\rho}_x\ddot{u}_x+\tilde{\rho}_{fx}\ddot{w}_x,\ \frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{zz}}{\partial z}=\tilde{\rho}_z\ddot{u}_z+\tilde{\rho}_{fz}\ddot{w}_z,$$
$$-\frac{\partial p}{\partial x}=\tilde{\rho}_{fx}\ddot{u}_x+\tilde{\mu}_x\ddot{w}_x,\qquad -\frac{\partial p}{\partial z}=\tilde{\rho}_{fz}\ddot{u}_z+\tilde{\mu}_z\ddot{w}_z, \tag{3}$$

where

$$\tilde{\rho}_z=\rho_1\theta_1+\rho_2\theta_2,\tilde{\rho}_{fz}=\rho_{f1}\theta_1+\rho_{f2}\theta_2,\tilde{\mu}=\tilde{m}_z+\tilde{b}_z/s,\tilde{m}_z=m_1\theta_1+m_2\theta_2,\tilde{b}_z=b_1\theta_1+b_2\theta_2,$$

$$\tilde{\rho}_x=\rho_1\theta_1+\rho_2\theta_2-\frac{\theta_1\theta_2(\rho_{f1}-\rho_{f2})^2}{m_2\theta_1+m_1\theta_2}+\frac{A}{s+D},\ \tilde{\rho}_{fx}=\frac{\rho_{f1}m_2\theta_1+\rho_{f2}m_1\theta_2}{m_2\theta_1+m_1\theta_2}+\frac{B}{s+D},$$
$$\tilde{\mu}_x=\tilde{m}_x+\frac{\tilde{b}_x}{s}+\frac{C}{s+D},\tilde{m}_x=\frac{m_1m_2}{m_2\theta_1+m_1\theta_2},\ \tilde{b}_x=\frac{b_1b_2}{b_2\theta_1+b_1\theta_2},\ D=\frac{\theta_1b_2+\theta_2b_1}{m_2\theta_1+m_1\theta_2},$$
$$A=\theta_1\theta_2(\theta_1b_2+\theta_2b_1)(\rho_{f1}-\rho_{f2})^2(\theta_1m_2+\theta_2m_1)^{-2}, \tag{4}$$
$$B=\theta_1\theta_2(\rho_{f1}-\rho_{f2})(m_1b_2-m_2b_1)(\theta_1m_2+\theta_2m_1)^{-2},$$
$$C=\theta_1\theta_2(m_1b_2-m_2b_1)^2(\theta_1b_2+\theta_2b_1)^{-1}(\theta_1m_2+\theta_2m_1)^{-2}.$$

One can see that averaging along x-axis is arithmetic one whereas along z-axis it is more complicated. This difference was not established in previous work [2] which corresponds to a partial case of (4) with $\tilde{\rho}_x = \tilde{\rho}_z$, $\tilde{\rho}_{fx} = \tilde{\rho}_{fz}$, $A = B = C = 0$. In our case $\tilde{\rho}_x$, $\tilde{\rho}_{fx}$ and $\tilde{\mu}_x$ become an operators. If we use relation

$$\frac{1}{D+s} f(t) = \int_0^t e^{-D(t-\tau)} f(\tau) d\tau, \tag{5}$$

then one can see that effective model contains not only viscous terms but also exponential relaxation kernels. Remarkable property of them is that always $AC = B^2$. Such kernels (only in $\tilde{\mu}$ terms) were successfully used by Carcione [3] for numerical modeling of wave propagation in anisotropic lossy porous media.

Effective model of finely layered medium (3), (4) may be considered as a generalized transversely isotropic Biot model with viscosity and relaxation. It is described by operators

$$\mu_z = m_z + \frac{b_z}{s}, \ \mu_x = m_x + \frac{b_z}{s} + \frac{C}{s+D}, \ \rho_x = \overline{\rho}_x + \frac{A}{s+D}, \ \rho_{fx} = \overline{\rho}_{fx} + \frac{B}{s+D}. \tag{6}$$

In order relaxation terms lead to attenuation the necessary and sufficient condition $AC \geq B^2$ should be satisfied .

If constituents itself originally have both viscous and relaxation terms than effective model of finely layered medium will be generalized transversely isotropic Biot medium with viscosity and several relaxation kernels.

The relaxation kernels may serve as a good description of attenuation and dispersion in porous reservoir rocks. To demonstrate their influence on attenuation let us consider a simple example when period consists of a hard layer ($\theta_1 = 0.98$, $\varepsilon = 0.15$, $\overline{\rho} = 2400\ kg/m^3$, $\alpha = 3.8$, $K_g = 38\ GPa$, $K = 16\ GPa$, $G = 15\ GPa$, $\kappa = 5 \times 10^{-15}\ m^2$) and soft one ($\theta_2 = 0.02$, $\varepsilon = 0.5$, $\overline{\rho} = 1825\ kg/m^3$, $\alpha = 1.8$, $K_g = 36\ GPa$, $K = 0.7\ GPa$, $G = 1.5 GPa$) which may describe a highly permeable fractures. Both layers are water saturated ($K_f = 2\ GPa$, $\eta = 10^{-3}\ kg/m\ sec$, $\rho_f = 1000\ kg/m^3$). In this case attenuation of usual P-waves along layers has two peaks (Figure 1, value above the curve is permeability of second layer). One of them is high frequency Biot peak and another is low-frequency peak caused by relaxation.

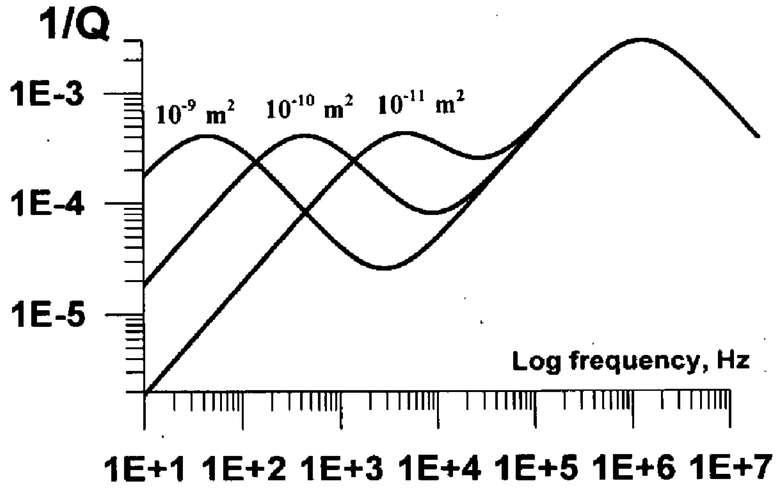

Figure 1.

References

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