Effective anisotropy due to thin VTI layers

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Summary

Seismic anisotropy observed in the field is a cumulative result of various mechanisms. Subsurface rocks often possess intrinsic anisotropy approximated by a transversely isotropic solid with a vertical symmetry axis (VTI). Alternatively, in the long-wavelength limit, "layer-induced" VTI anisotropy may arise even in a stack of thin isotropic constituent layers. In most practical cases these two effects occur simultaneously. We develop weak anisotropy and weak contrast approximations to understand the contributions of intrinsic and layer-induced anisotropy. When the contrast in elastic parameters between the constituents is small and their anisotropy is weak, then (to the first order) layering-induced anisotropy is insignificant whereas intrinsic anisotropy produces effective Thomsen parameters equal to the thickness-weighted average of the interval anisotropy parameters. This conclusion considerably simplifies upscaling of finely-layered VTI media because to find the effective Thomsen parameter ϵ (or δ or γ) one needs to know only constituents ϵ's (or δ's or γ's correspondingly).

For larger variation in the elastic properties, each anisotropic parameter may be approximated as the sum of two terms: one is the averaged intrinsic anisotropy and the other is a purely isotropic term related to fluctuations in the vertical interval velocities. The isotropic term has been extensively investigated in literature, and all previous conclusions may be directly applied to the more realistic VTI case considered in this study.

Introduction

Anisotropy caused by fine layering is often considered responsible for the differences between velocities obtained in sonic log and seismic experiments. Understanding the link between the two is becoming a must, especially in the current era of most wells being (highly) deviated.

Vertical sonic velocities may be upscaled according to Backus (1962) to deduce interval vertical velocities for seismic frequencies. There are, however, few experimental observations relating complete properties of fine layers to seismic anisotropy observed at seismic scale which is necessary if we would like to predict and explain reflection moveout for both P- and S-waves (Vernik and Fisher, 2001). Partly this is caused by the fact that fine layers themselves are anisotropic (e.g., shales). For a medium with two constituents (such as, sand and shale), the anisotropic parameters of both layers should be known to predict the properties of the effective compound (Backus, 1962). In addition, effective medium averaging is a nonlinear procedure which gives little insight into what to expect. A discussion was sparked by Thomsen's (1986) paper on how seismically measured anisotropy values relate to the properties of the thin layers and their intrinsic anisotropy (Levin, 1988). Although numerical schemes have existed for a long time, the physics behind them was not always clear. The situation is exemplified by Frank Levin's comment that "predicting the delta of a transversely isotropic solid from component delta's is not easy" (Levin, 1988).

This paper intends to improve understanding of how each Thomsen coefficient for effective transversely isotropic media with a vertical symmetry axis (VTI) depends on the VTI parameters of the individual constituents and the elastic contrasts between layers.

Exact averaging

Within our current abilities, most sedimentary rocks may be described by vertical transverse isotropy (VTI) (Thomsen, 1986). Each VTI constituent is defined by a stiffness matrix with five independent elements

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \tag{1}$$

where $c_{66} = (c_{11} - c_{12})/2$. The composite effective medium is also VTI according to the following averaging equations (Backus, 1962)

$$c_{11} = < c_{13}/c_{33} >^2 / < 1/c_{33} > - < c_{13}^2/c_{33} > + < c_{11} > , c_{12} = c_{11} - < c_{11} > + < c_{12} > , \tag{2}$$

$$c_{13} = < c_{13}/c_{33} > / < 1/c_{33} > , c_{33} = < 1/c_{33} >^{-1} , c_{44} = < 1/c_{44} >^{-1}, \tag{3}$$

where $c_{66} = (c_{11} - c_{12})/2 =< c_{66} >$. Here $< . >$ denotes the thickness-weighted average of corresponding parameters of individual constituents, for example, $< \alpha > = \phi_1 \alpha_1 + \phi_2 \alpha_2$ with ϕ_1 and $\phi_2 = 1 - \phi_1$ being their relative thicknesses.

Once the stiffnesses are obtained, they can be recast into Thomsen notation commonly used in reflection seismology

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}} \,, \qquad V_{S0} \equiv \sqrt{\frac{c_{44}}{\rho}} \,, \qquad \epsilon \equiv \frac{c_{11} - c_{33}}{2\, c_{33}} \,, \qquad \delta \equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2\, c_{33}\, (c_{33} - c_{44})} \,, \qquad \gamma \equiv \frac{c_{66} - c_{44}}{2\, c_{44}} \,, \quad (4)$$

where V_{P0} and V_{S0} are vertical velocities of P and S-waves, ρ is the density and ϵ, δ, and γ are the dimensionless Thomsen (1986) anisotropic parameters.

Weak-anisotropy and weak-contrast approximation

It is difficult to develop physical intuition for understanding and predicting the outcome of the exact equations (2)–(3) and (4). Even for *isotropic* constituents the results are not intuitive. Previous studies have attempted to derive some conclusions from numerical calculations and analysis of special cases considering only isotropic constituents. We will analyze the case of VTI constituent layers. We will first make two simplifying assumptions before proceeding with analysis:

1. The anisotropy of each constituent is weak. In the case of VTI layers this means that the Thomsen parameters are much smaller than unity ($|\epsilon| \ll 1$, $|\delta| \ll 1$ and $|\gamma| \ll 1$).
2. There is a weak contrast between the two constituents: $|\Delta c_{33}/\bar{c}_{33}| \ll 1$ and $|\Delta c_{44}/\bar{c}_{44}| \ll 1$.

The average stiffness $\bar{c}_{33} = 1/2(c_{33}^{(1)} + c_{33}^{(2)})$ and the difference $\Delta c_{33} = c_{33}^{(2)} - c_{33}^{(1)}$ are expressed as functions of stiffnesses of the first ($c_{33}^{(1)}$) and second constituent ($c_{33}^{(2)}$). Similar quantities are defined for the shear stiffness c_{44}. Note that the whole range $0 < c_{33}^{(2)}/c_{33}^{(1)} < \infty$ is mapped into $-2 < \Delta c_{33}/\bar{c}_{33} < 2$.

Here we are making weak-anisotropy and weak-contrast assumptions. In order to utilize them together, we also assume that the parameters ϵ, δ, γ, along with normalized jumps $\Delta c_{33}/\bar{c}_{33}$, $\Delta c_{44}/\bar{c}_{44}$, are small quantities of the same order. These assumptions are quite reasonable for many sedimentary sequences and are often used in anisotropic processing and AVO analysis.

Linearization of the effective stiffness matrix in these small quantities leads to the interesting result that the effective anisotropic parameters ϵ, δ, γ depend only on the corresponding Thomsen coefficients of the constituents:

$$\epsilon = < \epsilon > \,, \qquad \delta = < \delta > \,, \qquad \gamma = < \gamma > \,. \qquad (5)$$

Such results may be expected from the physics of wave propagation: in the limit of zero frequency, the effective media properties are independent of the order of layers for any number of constituents. For media with two constituents, this means that effective elastic properties should be independent of the signs of Δc_{33} and Δc_{44} and, thus, may not contain linear terms in contrasts.

Additional physical meaning of these results is most easily illustrated for Thomsen parameter γ. For isotropic constituents ($c_{66}^{(1)} = c_{44}^{(1)}$, $c_{66}^{(2)} = c_{44}^{(2)}$), the overall anisotropy γ is proportional to $< c_{44} > - < 1/c_{44} >^{-1}$. One can verify with simple algebra, that if averaged quantities are different by some small amount Δ, then to the first order in Δ, the geometric mean average is equivalent to the arithmetic mean average. Thus we conclude that:

- to the first order in elastic parameter contrasts isotropic layering does not produce effective anisotropy;
- effective anisotropy arises only if the constituents have intrinsic anisotropy;
- each effective anisotropic parameter is the thickness-weighted average of the corresponding parameters of the constituent layers.

To verify the accuracy of formulae (5), we compare them with the exact Thomsen parameters computed using equations (2)–(4). We focus only on predicting the effective Thomsen parameters ϵ, δ, γ because the vertical velocities of P- and S-waves can be easily computed from logs using exact equations (3)–(4). In the first two models (Table 1) we consider the extreme behavior of anti-correlated (V_{P0} increases while V_{S0} decreases) and correlated (both V_{P0} and V_{S0} increase) P- and S-velocities. Figures 1a–f demonstrate that, despite substantial values of anisotropic parameters and contrasts reaching 30%, the maximum error does not exceed 0.03. The parameters of model 3 are taken from the real case study by Vernik and Fisher (2001) who analyzed sand-shale sequences from the deepwater Gulf of Mexico. Figures 1g–i show that the maximum error while using our approximations does not exceed 0.015.

Case	$\frac{\Delta c_{33}}{\bar{c}_{33}}$	$\frac{\Delta c_{44}}{\bar{c}_{44}}$	ϵ_1	ϵ_2	δ_1	δ_2	γ_1	γ_2
1	30%	−30%	0.05	0.25	0.0	0.20	0.05	0.25
2	25%	30%	0.05	0.25	0.0	0.20	0.05	0.25
3	−45%	−27%	0.05	0.0	0.02	0.0	0.15	0.0

Table 1. Models used to test the accuracy of equations (5). First constituent in cases 1 and 2 has $V_{P0} = 3$ km/s, $V_{S0} = 1.5$ km/s, $\rho = 2.4$ g/cm^3, while in case 3 $V_{P0} = 3.2$ km/s, $V_{S0} = 1.55$ km/s, $\rho = 2.45$ g/cm^3.

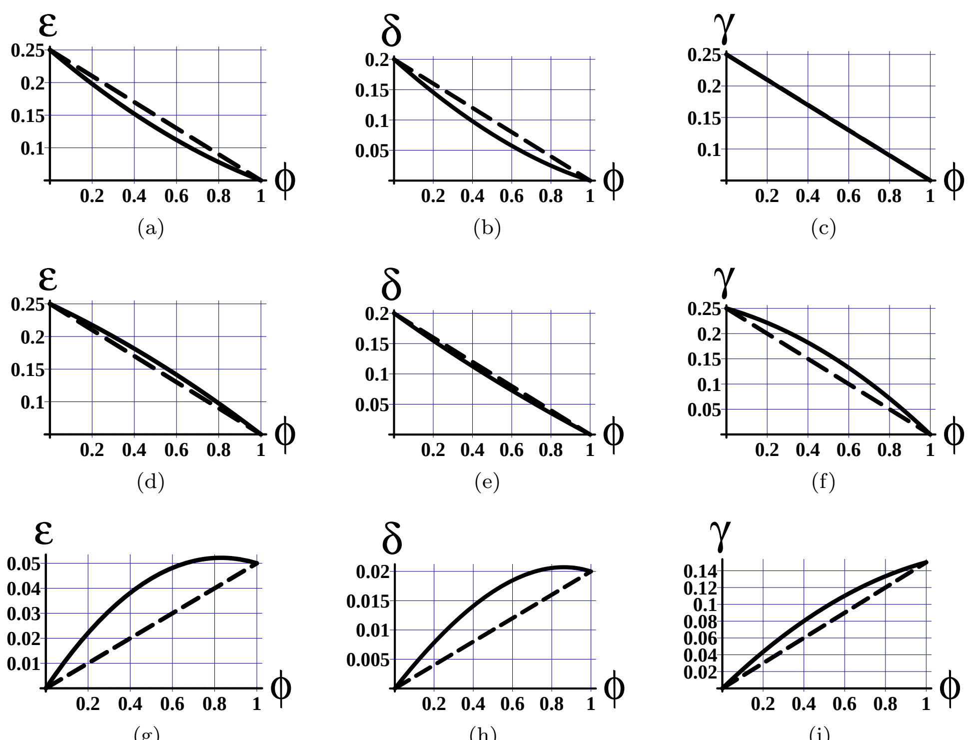

Fig. 1: Thomsen parameters of two-component VTI media as functions of the fraction of the first constituent $\phi = \phi_1$ ($\phi_2 = 1 - \phi_1$). Shown are the exact solutions (solid lines) and weak-anisotropy, weak-contrast approximations (dashed) [equations (5)]. Parameters are listed in Table 1: plots (a)–(c) correspond to case 1, whereas (d)–(f) to case 2, (g)–(i) to case 3.

Second-order approximation (stronger contrasts)

What happens if the property variation among the constituents is not small? The elastic parameter contrast lumps together the density and velocity contrasts according to the approximation $\Delta c_{33}/\bar{c}_{33} = \Delta\rho/\bar{\rho} + 2\Delta V_{P0}/\bar{V}_{P0}$. If, for example, we have a 20% density and 15% velocity contrast, this may result in $\Delta c_{33}/\bar{c}_{33}$ of about 50%. In this case the linearizations described above lead to erroneous predictions because the anisotropy caused by vertical heterogeneity will not be negligible. To include these effects we will obtain second-order approximations with respect to both intrinsic anisotropy and the contrasts in the elastic modulae. In so doing, we can gain useful insight into how effective anisotropy is influenced by intrinsic anisotropy, anisotropy induced by vertical heterogeneity and by their interaction. In the remainder of the section we analyze only Thomsen parameter δ, but similar analysis applies to other anisotropic coefficients.

Second-order approximation for the effective δ is represented by

$$\delta = <\delta> + \delta_{is} + \delta_{an} = <\delta> + 2\,\phi_1\,\phi_2\,\frac{\bar{c}_{44}}{\bar{c}_{33}}\left[\frac{\Delta c_{33}}{\bar{c}_{33}} - \frac{\Delta c_{44}}{\bar{c}_{44}}\right]\frac{\Delta c_{44}}{\bar{c}_{44}} - \frac{1}{2}\,\phi_1\,\phi_2\,\frac{(\Delta\delta)^2}{\left(1 - \frac{\bar{c}_{44}}{\bar{c}_{33}}\right)}\,, \tag{6}$$

where

- $<\delta>$ – the first-order term described before that depends on intrinsic anisotropy only,
- δ_{is} – the second-order "isotropic" term due to vertical heterogeneity obtained by replacing the VTI constituents by isotropic layers with the same vertical velocities,
- δ_{an} – the second-order term due to intrinsic anisotropy only.

For most subsurface boundaries we do not expect the contrast $|\Delta\delta|$ to be higher than $0.1 - 0.2$, which implies that third term $\delta_{an} < 0.025$ (for $\bar{c}_{44}/\bar{c}_{33} = 1/4$). Clearly, δ_{an} may be neglected in practice because it is smaller than $0.03 - 0.04$, which is minimum expected uncertainty in estimating interval δ from field data. Therefore we can use a simplified approximation,

$$\delta \approx <\delta> + \delta_{is}\,. \tag{7}$$

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The effective Thomsen δ is a simple sum of averaged intrinsic anisotropy $< \delta >$ and a purely isotropic contribution δ_{is} related to fluctuations in vertical elastic parameters. The last term in equation (7) has been extensively studied for isotropic components their conclusions may be readily transferred and applied to the case of VTI constituents.

Both our approximation (7) and the exact equation (2) show that no information about constituents ϵ's (or c_{11}'s) is required. The presence of the isotropic term can make the effective δ be smaller than $min(\delta_1, \delta_2)$ or larger than $max(\delta_1, \delta_2)$ as noted by Levin (1988). Therefore approximation (7) resolves the issue of predicting the effective δ for VTI constituents raised by Levin (1988) in his reply to the original Thomsen's paper (1986). Indeed, simple inspection of equations (6) and (7) leads to the following conclusions for two possible cases:

1. The second-order isotropic term δ_{is} is small compared to $< \delta >$. This happens when:
   - the contrasts are small enough to neglect the second-order term δ_{is}, which corresponds to the first-order approximation considered above.
   - $\frac{\Delta c_{44}}{\bar{c}_{44}} = 0$. In this case of a constant shear modulus, δ_{is} is always zero irrespective of how large $\Delta c_{33}/\bar{c}_{33}$ is.
   - $\frac{\Delta c_{44}}{\bar{c}_{44}} = \frac{\Delta c_{33}}{\bar{c}_{33}}$. This is equivalent to the case of a constant V_{S0}/V_{P0} or $\frac{c_{44}}{c_{33}}$. It can be recognized by acknowledging that to the first order $\frac{c_{44}^{(2)}}{c_{33}^{(2)}} = \frac{c_{44}^{(1)}}{c_{33}^{(1)}}(1 + \frac{\Delta c_{44}}{\bar{c}_{44}} - \frac{\Delta c_{33}}{\bar{c}_{33}})$. For this case again δ_{is} is always zero for any contrasts in the elastic modulae.

   For all three cases above we only need to know the constituent δ's to estimate the effective δ, and always $min(\delta_1, \delta_2) < \delta < max(\delta_1, \delta_2)$. This is a significant simplification compared to the exact equations (2)-(4) where three stiffnesses (or V_{P0}, V_{S0} and δ) have to be known to predict the effective δ.

2. The second-order isotropic term δ_{is} is comparable to $< \delta >$. This might happen only for sizeable contrasts between the constituents that also exhibit strong variations in the V_{S0}/V_{P0} ratio so that the term $\frac{\Delta c_{44}}{\bar{c}_{44}} - \frac{\Delta c_{33}}{\bar{c}_{33}}$ may become considerable.

Conclusions

We derived simple approximation for estimating effective seismic anisotropy caused by fine layering of VTI constituents. In the case when variation of vertical velocities is not large it predicts that effective Thomsen parameters are simply thickness-weighted average of the corresponding Thomsen parameters of the constituents. Such approximation is particularly attractive because it does not require any information on the vertical velocities apart from the fact that their variation is small (say, $\Delta c_{33}/\bar{c}_{33}$ and $\Delta c_{44}/\bar{c}_{44}$ are less than 30%). It is straightforward to prove that the linear approximation remains valid for any number of constituent layers.

The suggested approximation greatly simplifies the relationship between seismic and log/core elastic properties. The decoupling of anisotropic (Thomsen) parameters allows us to upscale/predict each of them independently with knowledge of only corresponding interval properties. The approximation makes it trivial to perform two important tasks: predicting effective seismic anisotropy needed for processing from core/lab measurements and inversion of seismically observed velocities/anisotropies for the properties of individual layers. A similar algorithm may be extremely useful in 4D workflow for unraveling time-lapse reservoir changes in anisotropy.

Approximations introduced in this study may also be useful to analyze VTI rocks permeated with sets of parallel fractures. For example, Bakulin et al. (2000; 2002) have shown that if set(s) of vertical fractures are added to VTI (finely-layered) background, then anisotropic parameters of effective orthorhombic media can be conveniently approximated as sum of intrinsic VTI anisotropy parameters and "isotropic" fracture-induced contributions. Complementing this result with current conclusions, we may further decompose effective orthorhombic coefficients to the sum of intrinsic Thomsen parameters of individual fine layers and fracture-induced contributions.

The second-order approximation becomes necessary when the contrasts in elastic modulae become significant (larger than 30-40%). Full second-order approximations do not give any computational advantage as they require the same input as the exact equations. However, they are instructive in analyzing the role of individual contributions into the overall anisotropy.

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