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BIOT DESCRIPTION OF THINLY LAYERED POROUS MEDIUM WITH IMPERMEABLE SOLID LAYERS

LEV MOLOTKOV[1] and ANDREY BAKULIN[2]

[1] *Russian Academy of Sciences, St. Petersburg Branch of Steklov, Mathematical Institute, 27 Fontanka, 191011 St. Petersburg, Russia*

[2] *St. Petersburg State University*

Oil and gas collecting rocks may have strong anisotropy of filtration properties. This anisotropy is qualitatively accounted by incorporating set of fractures in background solid rock. However real rocks also have significant effective porosity [1]. In order to combine both effects we consider a porous rock which is impermeable in one direction whereas it preserves fluid mobility in perpendicular one. This model could be applied to fractured reservoir rocks having strong anisotropy of filtration properties. This rock is schematically presented on Fig. 1 where alternating Biot and solid elastic layers occupy a region $0 \leq z \leq H$. Homogeneous and isotropic Biot layers are described by equations

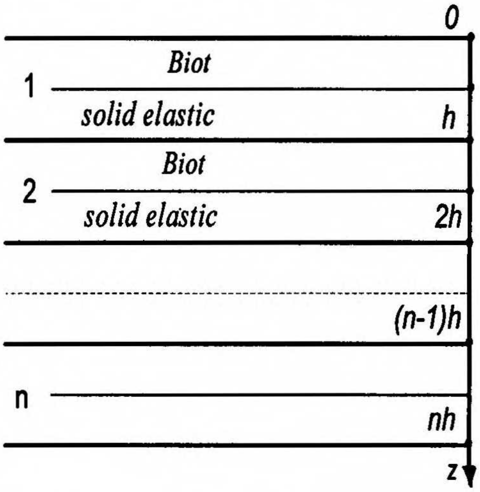

Figure 1

$$\tau_{xx} = P\frac{\partial u_x}{\partial x} + F\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$\tau_{zz} = F\frac{\partial u_x}{\partial x} + P\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$-p = M\frac{\partial u_x}{\partial x} + M\frac{\partial u_z}{\partial z} + R(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$\tau_{xz} = L(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}), \qquad (1)$$
$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \bar{\rho}\ddot{u}_x + \rho_f \ddot{w}_x,$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = \bar{\rho}\ddot{u}_z + \rho_f \ddot{w}_z,$$
$$-\frac{\partial p}{\partial x} = \rho_f \ddot{u}_x + m\ddot{w}_x, \qquad (2)$$
$$-\frac{\partial p}{\partial z} = \rho_f \ddot{u}_z + m\ddot{w}_z,$$

where u_x, u_z - displacements of solid phase, w_x, w_z - displacements of fluid relative to solid, τ_{xx}, τ_{zz}, τ_{xz} – total bulk stresses in porous medium, p - fluid pressure, $\bar{\rho}$ - volume averaged density, ρ_f - fluid density, $m = \rho_f\alpha/\varepsilon$, α – tortuosity of pore space, ε – porosity. Solid elastic layers are also homogeneous and isotropic with Lame parameters λ, μ and density ρ. Displacements U_x, U_z and stresses t_{xx}, t_{zz},t_{xz} in solid layers are described by equations of theory of elasticity. Boundary conditions on solid-porous interfaces are $u_x = U_x$, $u_z = U_z$, $\tau_{xx} = t_{xx}$, $\tau_{zz} = t_{zz}$, $w_z = 0$.

Construction of effective long-wave equivalent medium is done by the method of matrix averaging developed by L.Molotkov [2]: in the full wave field build by matrix method we do a passage to the limit when number of layers $n \to \infty$, thickness of period $h \to 0$ whereas total thickness of stack $H = nh$ is kept constant together with the portions of first θ_1 and second θ_2 layers.

The effective model is described by equations

$$\tau_{xx} = \tilde{P}\frac{\partial u_x}{\partial x} + \tilde{F}\frac{\partial u_z}{\partial z} + \tilde{M}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{zz} = \tilde{F}\frac{\partial u_x}{\partial x} + \tilde{C}\frac{\partial u_z}{\partial z} + \tilde{Q}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$-p = \tilde{M}\frac{\partial u_x}{\partial x} + \tilde{Q}\frac{\partial u_z}{\partial z} + \tilde{R}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{xz} = \tilde{L}(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}), \qquad (3)$$

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \tilde{\bar{\rho}}\ddot{u}_x + \rho_f \ddot{w}_x, \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = \tilde{\bar{\rho}}\ddot{u}_z,$$
$$-\frac{\partial p}{\partial x} = \rho_f \ddot{u}_x + m_x \ddot{w}_x, \tag{4}$$

$$\tilde{P} = \theta_1 \left[(P-F)(PR+RF-M^2+M\tilde{M}) + (RF-M^2)\tilde{F}\right] \Big/ (PR-M^2) +$$
$$+\ \theta_2[4\mu(\lambda+\mu)+\lambda\tilde{F}] \Big/ (\lambda+2\mu), \quad \tilde{F} = \triangle[F(\lambda+2\mu)\theta_1 + P\lambda\theta_2],$$
$$\tilde{M} = M\triangle[(\lambda+2\mu)\theta_1 + (\lambda+P-F)\theta_2], \ \tilde{Q} = M\triangle(\lambda+2\mu), \ \tilde{C} = P\triangle(\lambda+2\mu),$$
$$\tilde{R} = \triangle[R(\lambda+2\mu)\theta_1 + (PR-M^2)\theta_2]/\,\theta_1, \ \tilde{L} = 1/\,(\theta_1/L + \theta_2/\mu)\,,$$
$$\triangle = 1/[(\lambda+2\mu)\theta_1 + P\theta_2], \tilde{\bar{\rho}} = \theta_1\bar{\rho} + \theta_2\rho, m_x = m/\theta_1. \tag{5}$$

According to Eqs.(4),(5), effective model is a special partial case of transversaly isotropic Biot model which has $m_z = \infty$, $w_z = 0$ and the last equation containing $\partial p/\partial z$ is rejected as uncertain.

Investigation of Eqs.(4),(5) gives a possibility to build a wave fronts from a point source at the origin. Main feature of this model is that the one front is triangular. It means that while three waves propagate along x-axis, only two of them survive along z-axis. Second P-wave is suppressed along z-axis because of absence of relative fluid-solid motion in this direction. Similar wave with triangle front takes place in effective model of alternating fluid/solid layers [2]. Velocities are along x-axis

$$v_{1,2}^2 = \frac{(\tilde{R}\tilde{\bar{\rho}} + \tilde{P}m_x - 2\tilde{M}\rho_f) \pm \sqrt{(\tilde{R}\tilde{\bar{\rho}} + \tilde{P}m_x - 2\tilde{M}\rho_f)^2 - 4(\tilde{P}\tilde{R} - \tilde{M}^2)(\tilde{\bar{\rho}}m_x - \rho_f^2)}}{2(\tilde{\bar{\rho}}m_x - \rho_f^2)}$$
$$v_3^2 = \tilde{L}/\tilde{\bar{\rho}}, \quad v_4^2 = (\tilde{C}\tilde{R} - \tilde{Q}^2) \Big/ (\tilde{C}m_x) \tag{6}$$

and along z-axis $v_5^2 = \tilde{C}/\tilde{\bar{\rho}}$, $v_6^2 = \tilde{L}m_x/(\tilde{\bar{\rho}}m_x - \rho_f^2)$. At the point v_4 on x-axis front of second P-wave is tangent to the axis. Velocity v_4 is a velocity in a plate made of transversaly isotropic Biot medium and covered by thin impermeable film so that fluid can not move in and out of pores. Examples of fronts are given on Fig.2 and 3, where porosity $\varepsilon = 0.2$ and $\tilde{\bar{\rho}} = 2.36\ g/cm^3$, $\rho_f = 1\ g/cm^3$, $m_x = 15\ g/cm^3$. Elastic parameters for Fig.2 are $\tilde{P} = 50\ GPa$, $\tilde{C} = 20\ GPa$, $\tilde{F} = 8\ GPa$, $\tilde{Q} = 12\ GPa$, $\tilde{M} = 10\ GPa$, $\tilde{R} = 8\ GPa$, $\tilde{L} = 4\ GPa$, while for Fig.3 – $\tilde{P} = 33.8\ GPa$, $\tilde{C} = 15\ GPa$, $\tilde{F} = 6.9\ GPa$, $\tilde{M} = \tilde{Q} = 7.5\ GPa$, $\tilde{R} = 30\ GPa$, $\tilde{L} = 1\ GPa$. On Fig. 2 triangle front corresponds to second P-wave while on Fig. 3 transition of $P2$ to S-wave takes place on the same front.

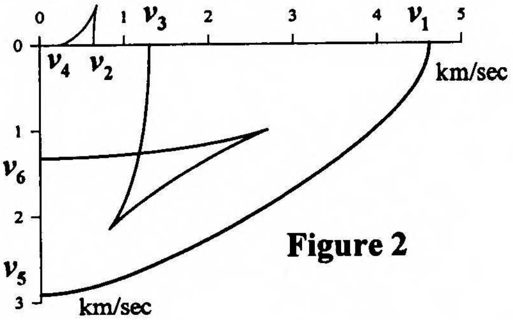

Figure 2

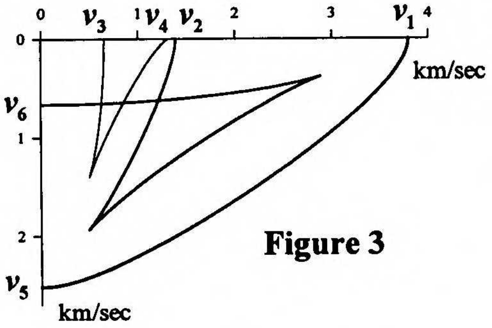

Figure 3

References

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