# P085 APPLICATION OF COMPLEX BIOT DENSITIES FOR THE DESCRIPTION OF ATTENUATION AND DISPERSION IN POROUS ROCKS

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Attenuation in porous rocks is still poorly understood although it both affects the wavefields and contains additional information on rock properties. Dispersion and attenuation in artificial sandstones are well described by Biot type of loss with corrections for dynamic permeability [1] but it seems that in real rocks other mechanisms overpower this effect [2]. We feel that it is important to find a simple phenomenological description of attenuation which can be used in numerical modeling and inversion.

The idea is to incorporate the attenuation in Biot equations by making some terms complex. In elasticity it is usually elastic moduli which are made complex and frequency dependent. However the number of elastic moduli in isotropic poroelasticity is four whereas in transversely isotropic case it is eight. That is why we suggest to put the attenuation in densities in the same fashion as it was did by M.Biot. There are only three densities for isotropic case and four for transversely isotropic case. Original Biot equations are formally the same

$$\begin{aligned}\tau_{xx} &= P\frac{\partial u_x}{\partial x} + F\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{zz} = F\frac{\partial u_x}{\partial x} + P\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\\ -p &= M\frac{\partial u_x}{\partial x} + M\frac{\partial u_z}{\partial z} + R(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{xz} = L(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}),\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= \overline{\rho}\ddot{u}_x + \tilde{\rho}_f \ddot{w}_x, & \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} &= \overline{\rho}\ddot{u}_z + \tilde{\rho}_f \ddot{w}_z,\\ -\frac{\partial p}{\partial x} &= \tilde{\rho}_f \ddot{u}_x + \tilde{m}\ddot{w}_x, & -\frac{\partial p}{\partial z} &= \tilde{\rho}_f \ddot{u}_z + \tilde{m}\ddot{w}_z,\end{aligned} \tag{2}$$

where $u_x$, $u_z$ - displacements in solid phase, $w_x$, $w_z$ - displacements of fluid relative to skeleton, $\tau_{xx}$, $\tau_{zz}$, $\tau_{xz}$ – total bulk stresses in porous medium, $p$ - fluid pressure. However all the densities are now complex functions which in frequency domain are

$$\overline{\rho} = \rho + \sum_{j=1}^{n} \frac{A_j}{D_j - i\omega},\quad \tilde{\rho}_f = \rho_f + \sum_{j=1}^{n} \frac{B_j}{D_j - i\omega},\quad \tilde{m} = m + i\frac{b}{\omega} + \sum_{j=1}^{n} \frac{C_j}{D_j - i\omega}, \tag{3}$$

where $\rho$ - volume averaged density, $\rho_f$ - fluid density; $m = \rho_f \alpha/\varepsilon$, $\alpha$ – tortuosity of pore space, $\varepsilon$ – porosity; $b = \eta/\kappa$, $\eta$ - fluid viscosity, $\kappa$ - permeability.

In order imaginary parts of densities lead to attenuation the following conditions should be satisfied

$$\rho > 0,\ \rho_f > 0,\ m > 0,\ \rho m - \rho_f^2 \geq 0,\ A_i \geq 0,\ C_i \geq 0,\ D_i \geq 0,\ A_i C_i - B_i^2 \geq 0. \tag{4}$$

Terms containing $A$, $B$, $C$ and $D$ naturally appear after averaging of finely layered poroelastic medium with Biot type of loss inside constituents [3]. Low-frequency velocities are the same as in case of only Biot attenuation ($A = B = C = 0$). However overall attenuation is changed and at low-frequency for $n = 1$ it is

$$Q_P^{-1} = \omega \left[ \frac{A}{\rho D^2} + \frac{\rho_f^2}{b\rho} \left( 1 - \frac{M\rho}{P\rho_f} \right)^2 \right], \quad Q_S^{-1} = \omega \left[ \frac{A}{\rho D^2} + \frac{\rho_f^2}{b\rho} \right]. \tag{5}$$

One can see that second term in inverse $Q$ is exactly the Biot loss whereas the first one is additional loss due to $A$ and $D$. In general (3) leads to an additional peak in inverse $Q$. By changing the parameters it is easy to fit both the level and the position of this peak.

We demonstrate it by very simple example with $n = 1$ and $B = C = 0$. So now we have to fit only 2 parameters $A$ and $D$. On the Figure 1 one can see the attenuation and dispersion curves for a fast P-wave in a water-saturated sandstone with $A = 9.5 \times 10^7\ kg/m^3sec$, $D = 4.1 \times 10^5\ 1/sec$ and other parameters $\varepsilon = 0.15$, $\overline{\rho} = 2400\ kg/m^3$, $\rho_f = 1000\ kg/m^3$, $\alpha = 3.8$, $P = 39\ GPa$, $F = 10.2\ GPa$, $R = 11.6\ GPa$, $L = 14.6\ GPa$, $\kappa = 5 \times 10^{-15}\ m^2$, $\eta = 10^{-3}\ kg/m\ sec$. These curves are almost the same as the predictions of squirt flow model by Dvorkin and Nur ([2], compare to their Figures 2 and 3 for 5 mD) which are believed to be typical for a porous rocks.

Numerical implementation of suggested attenuation is straightforward. It was successfully demonstrated by Carcione [4] however he used only additional $C$-term in density $\tilde{m}$ which corresponds to $A = B = 0$ in our case. The $C$-term does not affect the low-frequency attenuation (5). Moreover numerical simulation shows that in general $C$-term could not even provide the high value of the additional peak whereas $A$-term is free of all mentioned shortcomings. To account for an anisotropic attenuation in rocks one just should choose different kernels for densities along $x-$ and $z$-axes.

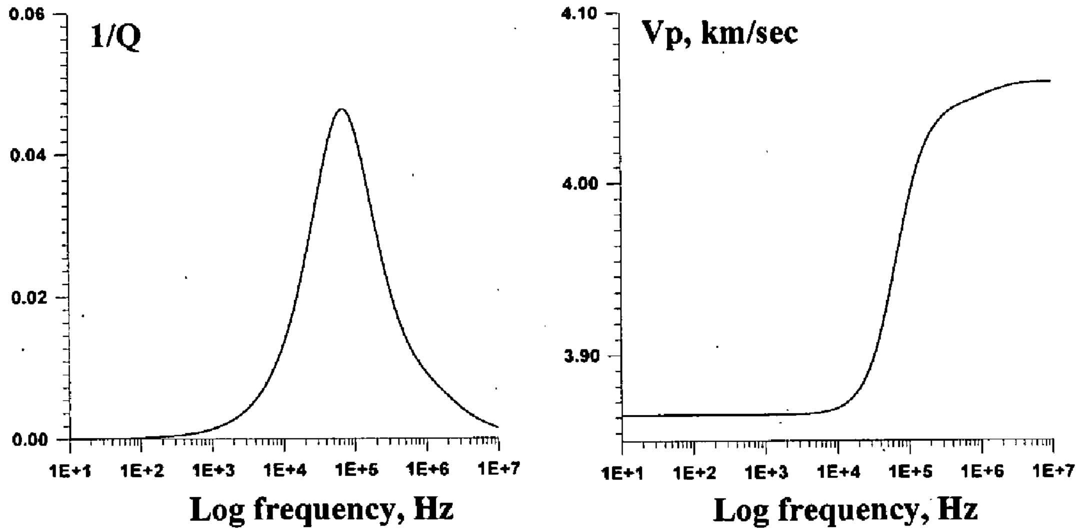


Figure 1. Calculated attenuation and dispersion curve.

**References**


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