P055 GENERALIZED ANISOTROPIC BIOT MODEL AS EFFECTIVE MODEL OF STRATIFIED POROELASTIC MEDIUM

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It is well-known that oil and gas reservoirs rocks are often anisotropic. Mostly this anisotropy is due to fine layering and fracturing. When constituents of interleaving layers are elastic the effective medium is also elastic anisotropic. Anisotropic porous media are assumed to be well described by Biot model with anisotropic skeleton, tortuosity and permeability. However, Biot model does not describe the situation when constituents are themselves porous. It may be caused for example by interleaving of fine-grained and large-grained sandstones which may differ by permeability, tortuosity and frame moduli [1]. This layering highly influence the permeability and tortuosity anisotropy which may be 2-3 orders [1, 2]. Here we present the generalization of Biot model which accounts for such effects.

We consider periodic medium each period of thickness h consists of two isotropic poroelastic Biot layers described by equations

$$\tau_{xx} = P\frac{\partial u_x}{\partial x} + F\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{zz} = F\frac{\partial u_x}{\partial x} + P\frac{\partial u_z}{\partial z} + M(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$-p = M\frac{\partial u_x}{\partial x} + M\frac{\partial u_z}{\partial z} + R(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{xz} = L(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}), \quad (1)$$

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \overline{\rho}\ddot{u}_x + \rho_f\ddot{w}_x,\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} = \overline{\rho}\ddot{u}_z + \rho_f\ddot{w}_z,$$
$$-\frac{\partial p}{\partial x} = \rho_f\ddot{u}_x + m\ddot{w}_x,\qquad -\frac{\partial p}{\partial z} = \rho_f\ddot{u}_z + m\ddot{w}_z, \quad (2)$$

where u_x, u_z - displacements in solid phase, w_x, w_z - displacements of fluid relative to skeleton, τ_{xx}, τ_{zz}, τ_{xz} – total bulk stresses in porous medium, p - fluid pressure, $\overline{\rho}$ - volume averaged density, ρ_f - fluid density, $m = \rho_f\alpha/\varepsilon$, α – tortuosity of pore space, ε – porosity. Each constituent in period has a portion θ_1 and θ_2 ($\theta_1+\theta_2 = 1$). This medium consisting total of n periods occupies region from 0 to $H = nh$ along vertical z-axis. On every interface quantities u_x, u_z, w_z, τ_{zz}, τ_{xz} and p are continuous.

For defined periodic media we constructed effective models (constitutive equations) by means of method of matrix averaging developed by L.Molotkov[3]. Transition from layered medium to the effective homogeneous medium is done by passage to the limit in the full wave field when $n \to \infty$, $h \to 0$, $H = nh = const$. Effective model in case of porous-porous layers is described by equations

$$\tau_{xx} = \tilde{P}\frac{\partial u_x}{\partial x} + \tilde{F}\frac{\partial u_z}{\partial z} + \tilde{M}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{zz} = \tilde{F}\frac{\partial u_x}{\partial x} + \tilde{C}\frac{\partial u_z}{\partial z} + \tilde{Q}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),$$
$$-p = \tilde{M}\frac{\partial u_x}{\partial x} + \tilde{Q}\frac{\partial u_z}{\partial z} + \tilde{R}(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}),\ \tau_{xz} = \tilde{L}(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}), \quad (3)$$

$$\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=\overline{\rho}_x\ddot{u}_x+\rho_{fx}\ddot{w}_x,\quad \frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{zz}}{\partial z}=\overline{\rho}_z\ddot{u}_z+\rho_{fz}\ddot{w}_z,$$
$$-\frac{\partial p}{\partial x}=\rho_{fx}\ddot{u}_x+m_x\ddot{w}_x,\qquad -\frac{\partial p}{\partial z}=\rho_{fz}\ddot{u}_z+m_z\ddot{w}_z. \tag{4}$$

where

$$\overline{\rho}_z=\overline{\overline{\rho}},\overline{\rho}_x=\overline{\left(\frac{\overline{\rho}m-\rho_f^2}{m}\right)}+\overline{\left(\frac{\rho_f}{m}\right)}^2\overline{\left(\frac{1}{m}\right)}^{-1},$$
$$\overline{\rho}_{fx}=\overline{\left(\frac{\rho_f}{m}\right)}\overline{\left(\frac{1}{m}\right)}^{-1},\overline{\rho}_{fz}=\overline{\rho_f},m_x=\overline{\left(\frac{1}{m}\right)}^{-1},m_z=\overline{m}; \tag{5}$$

$$\tilde{P}=\overline{\left(\frac{FR-M^2}{d}\right)}\tilde{F}+\overline{\left(\frac{2LM}{d}\right)}\tilde{M}+\overline{\left(\frac{D}{d}\right)},\ \tilde{C}=\frac{1}{f}\overline{\left(\frac{P}{d}\right)},\ \tilde{Q}=\frac{1}{f}\overline{\left(\frac{M}{d}\right)},\ \tilde{R}=\frac{1}{f}\overline{\left(\frac{R}{d}\right)},$$
$$\tilde{F}=\frac{1}{f}\left\{\overline{\left(\frac{P}{d}\right)}\,\overline{\left(\frac{FR-M^2}{d}\right)}+\overline{\left(\frac{M}{d}\right)}\,\overline{\left(\frac{2LM}{d}\right)}\right\},f=\overline{\left(\frac{P}{d}\right)}\,\overline{\left(\frac{R}{d}\right)}-\overline{\left(\frac{M}{d}\right)}^2,\tilde{L}=\overline{\left(\frac{1}{L}\right)}^{-1},$$
$$\tilde{M}=\frac{1}{f}\left\{\overline{\left(\frac{M}{d}\right)}\,\overline{\left(\frac{FR-M^2}{d}\right)}+\overline{\left(\frac{R}{d}\right)}\,\overline{\left(\frac{2LM}{d}\right)}\right\},d=PR-M^2,D=\begin{vmatrix}P&F&M\\F&P&M\\M&M&R\end{vmatrix}. \tag{6}$$

We call this generalized transversaly isotropic Biot model because $\overline{\rho}_x\neq\overline{\rho}_z$ and $\overline{\rho}_{fx}\neq\overline{\rho}_{fz}$. This generalization is due to the fact that total and fluid densities are averaged by different manner along x and z axes and become tensors. The total density is tensor even if two layers differ only by saturating fluid density. The same differences were also in case of effective model of stratified fluid-fluid medium [3]. If fluid density is constant in period ($\rho_{f1}=\rho_{f2}$) then $\overline{\rho}_x=\overline{\rho}_z$, $\overline{\rho}_{fx}=\overline{\rho}_{fz}$ and effective model becomes ordinary transversaly isotropic Biot medium. Examples of wavefronts are given on Fig.1 ($\tilde{P}=50\ GPa$, $\tilde{C}=20\ GPa$, $\tilde{F}=8\ GPa$, $\tilde{M}=\tilde{Q}=10\ GPa$, $\tilde{R}=10\ GPa$, $\tilde{L}=4\ GPa$, $\overline{\rho}_x=2.36\ g/cm^3$, $\overline{\rho}_z=3.54\ g/cm^3$, $\overline{\rho}_{fx}=1\ g/cm^3$, $\overline{\rho}_{fz}=1.5\ g/cm^3$, $m_x=15\ g/cm^3$, $m_z=45\ g/cm^3$). For the velocities in generalized transversaly isotropic Biot model along axes one can write the rather simple expressions.

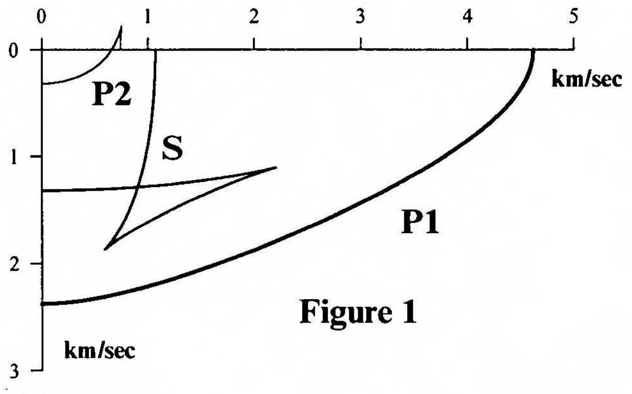

Figure 1

Derived expressions for effective densities (4) and moduli (5) give direct possibility to do a passage to the limit when second layer in period is thin and soft. In addition thin layer may be taken permeable or impermeable. It corresponds to porous medium with set of fractures.

References

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