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## Summary

The simplest effective model of a formation containing a single fracture system is transversely isotropic with a horizontal symmetry axis (HTI medium). Reflection seismic signatures over HTI media can be concisely described by Thomsen-type anisotropic parameters  $\epsilon^{(V)}$ ,  $\delta^{(V)}$  and  $\gamma^{(V)}$ . Here, we use effective medium theory to study the dependence of the anisotropic parameters on the physical properties of the crack system, as well as to devise a fracture-characterization procedure operating with surface seismic data.

Simple expressions, linearized in the crack density, show that there is a substantial difference between the values of the anisotropic coefficients for isolated fluid-filled and dry (gas-filled) cracks. While the dry-crack model is close to elliptical ( $\epsilon^{(V)} \approx \delta^{(V)}$ ), for fluid-filled cracks  $\epsilon^{(V)} \approx 0$ , and the absolute value of  $\delta^{(V)}$  for typical  $V_S/V_P$  ratios in the background is close to the crack density. For purposes of estimating the density and content of the cracks, it is convenient to invert the anisotropic parameters for the normal and tangential weaknesses of the crack system. We show that azimuthally dependent  $P$ -wave traveltimes (in some cases, combined with prestack amplitudes) provide enough information for this inversion procedure, if an estimate of the  $V_S/V_P$  ratio is available.

## Introduction

Seismic detection of subsurface fractures has important applications in characterization of fractured reservoirs. However, most existing studies concentrate on analysis of time delays or reflection amplitudes of split shear waves at near-vertical incidence. While such measurements make it possible to find the fracture orientation and intensity (or crack density) of vertical fractures, they are not sensitive to the fluid content of the fracture network. Recently, it was demonstrated that the azimuthal dependence of  $P$ -wave signatures has the potential of determining both the crack orientation (e.g., Corrigan et al., 1996) and crack density (Tsvankin, 1997). Here, extending the results of Tsvankin (1997) and Contreras et al. (1999), we show that a complete fracture characterization procedure (that includes evaluation of the fluid content) can be based on conventional  $P$ -wave data recorded in "wide-azimuth" 3-D surveys.

## Basic model of fractured media

To obtain the effective anisotropic parameters of fractured media, we use the theory of linear slip (Schoenberg and Sayers, 1995; Bakulin and Molotkov, 1998). One set of vertical rotationally invariant fractures embedded in isotropic host rock yields an HTI model with the following elastic parameters:

$$\begin{aligned} c_{11} &= (\lambda + 2\mu)(1 - \Delta_N), & c_{13} &= \lambda(1 - \Delta_N), \\ c_{33} &= (\lambda + 2\mu)\left(1 - \frac{\lambda^2}{(\lambda + 2\mu)^2} \Delta_N\right), & c_{44} &= \mu, & c_{55} &= \mu(1 - \Delta_T), \end{aligned} \quad (1)$$

where  $\lambda$  and  $\mu$  are the Lamé parameters of the host rock, and  $\Delta_N$  and  $\Delta_T$  are two dimensionless parameters controlled by the compliance of the fractures (Schoenberg and Sayers, 1995), which can be called normal and shear weaknesses (Bakulin and Molotkov, 1998). Each weakness varies from 0 to 1, with zero values of both  $\Delta_N$  and  $\Delta_T$  for unfractured media and unity corresponding to the total slip in either normal or tangential direction to the fractures.

In principle, other effective models of rock with thin parallel fractures (cracks) can be treated as special cases of the linear-slip theory. The main significance of these models is in providing a useful insight into the dependence of the fracture weaknesses on the microstructural (physical) parameters. For example, in the case of dry (gas-filled) penny-shaped cracks (Hudson, 1981), the weaknesses are given by

$$\Delta_N^{\text{dry}} = \frac{4e}{3g(1-g)} \quad \text{and} \quad \Delta_T^{\text{dry}} = \frac{16e}{3(3-2g)}. \quad (2)$$

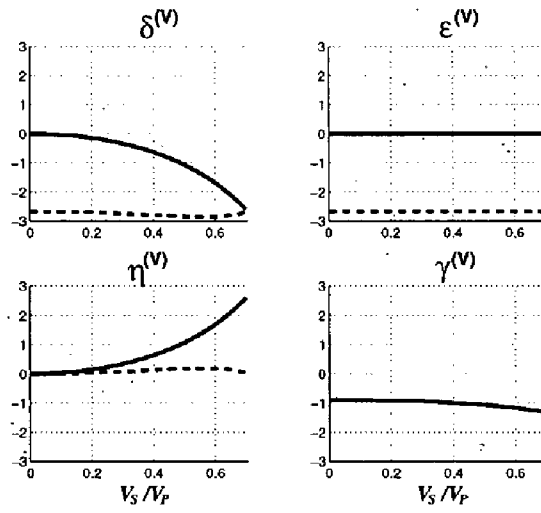


FIG. 1. Anisotropic coefficients for effective HTI media due to isolated vertical fluid-filled (solid) and dry (dashed) penny-shaped cracks in isotropic background. The vertical scale is in the units of the crack density  $e$ .

If the cracks are completely filled with fluid,

$$\Delta_N^{\text{wet}} = 0 \quad \text{and} \quad \Delta_T^{\text{wet}} = \frac{16e}{3(3-2g)}. \quad (3)$$

Here  $e$  is the crack density (the number of cracks per unit volume multiplied with their mean cubed diameter) and  $g = V_P^2/V_S^2$  is the squared ratio of the background velocities. For cracks partially saturated with fluid or hydraulically connected to pore space (Thomsen, 1995),  $\Delta_T$  remains the same, while  $\Delta_N$  takes an intermediate value between  $\Delta_N^{\text{dry}}$  and  $\Delta_N^{\text{wet}} = 0$ .

### Anisotropic coefficients of fractured rocks

Seismic signatures in HTI media are most conveniently described by the dimensionless anisotropic parameters  $\epsilon^{(V)}$ ,  $\delta^{(V)}$  and  $\gamma^{(V)}$  introduced in Tsvankin (1997) by analogy with the well-known Thomsen coefficients for vertical transverse isotropy. Expressing these parameters in terms of fracture weaknesses and linearizing the exact equations with respect to  $\Delta_N$  and  $\Delta_T$  yields

$$\begin{aligned} \gamma^{(V)} &= \frac{c_{66} - c_{44}}{2c_{44}} = -\frac{\Delta_T}{2}, & \delta^{(V)} &= \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})} \approx -2g[(1-2g)\Delta_N + \Delta_T], \\ \epsilon^{(V)} &= \frac{c_{11} - c_{33}}{2c_{33}} \approx -2g(1-g)\Delta_N, & \eta^{(V)} &\equiv \frac{\epsilon^{(V)} - \delta^{(V)}}{1 + 2\delta^{(V)}} \approx 2g(\Delta_T - g\Delta_N). \end{aligned} \quad (4)$$

According to equations (4), for vertical parallel fractures  $\epsilon^{(V)} \leq 0$ ,  $\delta^{(V)} \leq 0$ ,  $\gamma^{(V)} \leq 0$ , and  $\eta^{(V)} \geq 0$ . Since the effective HTI medium is fully defined by four quantities ( $\lambda$ ,  $\mu$ ,  $\Delta_N$ , and  $\Delta_T$ ), the anisotropic coefficients  $\epsilon^{(V)}$ ,  $\delta^{(V)}$ , and  $\gamma^{(V)}$  are not independent. For example, if we know  $\epsilon^{(V)}$  and  $\delta^{(V)}$ , the shear-wave splitting parameter  $\gamma^{(V)}$  can be obtained from the following linearized expression (the exact formula is given in Tsvankin, 1997):

$$\gamma^{(V)} = \frac{1}{4g} \left( \delta^{(V)} - \epsilon^{(V)} \frac{1-2g}{1-g} \right). \quad (5)$$

Thus, for HTI media due to vertical fractures, the shear-wave splitting parameter  $\gamma^{(V)}$  can be estimated from  $P$ -wave traveltime data which allow one to obtain  $\epsilon^{(V)}$  and  $\delta^{(V)}$  (Tsvankin, 1997; Contreras et al., 1999). Equations (4) can be rewritten directly in terms of crack density for isolated dry and fluid-filled cracks using Hudson's (1981) theory.

### Influence of fluid content on the anisotropic coefficients

To evaluate the influence of fluid content on the anisotropic coefficients, we use Hudson's (1981) model that provides the dependence of the weaknesses  $\Delta_N$  and  $\Delta_T$  on the crack density, saturation and the background velocity ratio  $g$  [equations (2)–(3)]. Substituting equations (2)–(3) into equations (4), we obtain the anisotropic coefficients displayed in Figure 1.

As expected, the presence of fluid does not influence shear-wave splitting parameter  $\gamma^{(V)}$ . In contrast, the values of both  $\epsilon^{(V)}$  and  $\delta^{(V)}$  for fluid-filled and dry cracks are substantially different. The parameter  $\epsilon^{(V)}$  is a function of only one weakness ( $\Delta_N$ ), which is responsible for the jump of the normal displacement across the crack face and, therefore, is strongly dependent on the fluid bulk modulus. The difference between  $\epsilon^{(V)}$  for dry and fluid-filled cracks remains the same ( $8e/3$ ) for any  $V_S/V_P$  ratio. For the parameter  $\delta^{(V)}$ , the influence of the crack infill rapidly decreases with the  $V_S/V_P$  ratio. If  $V_S/V_P$  takes a typical value 0.5, the absolute value of  $\delta^{(V)}$  for dry cracks is still almost three times greater than that for fluid-filled cracks. For fluid-filled cracks and  $V_S/V_P \approx 0.5$ ,  $\delta^{(V)} \approx \gamma^{(V)} \approx -e$ . For dry cracks, the "anellipticity" parameter  $\eta^{(V)}$  is close to zero (e.g., the medium is elliptically anisotropic), while for fluid-filled cracks  $\eta^{(V)}$  is always positive. The anisotropic parameters for models with hydraulically interconnected cracks and pores (Thomsen, 1995) or with partially saturated cracks always lie between the values for dry and fluid-filled isolated cracks plotted in Figure 1.

### Estimation of fracture parameters

The above analysis of anisotropic coefficients suggests that the weaknesses  $\Delta_N$  and  $\Delta_T$  can be found from seismic data if any *two* anisotropic coefficients (e.g.,  $\epsilon^{(V)}$  and  $\delta^{(V)}$ ) and an estimate of the  $V_S/V_P$  ratio are available. While the tangential weakness  $\Delta_T$  depends only on crack density [equations (2) and (3)], the normal weakness  $\Delta_N$  is strongly influenced by fluid content [compare equations (2) and (3) for  $\Delta_N$ ], thus making it possible to discriminate between wet and dry cracks and, in general, estimate fluid saturation.

Solving the first two equations (4) for the fracture weaknesses, we obtain

$$\Delta_N = -\frac{\epsilon^{(V)}}{2g(1-g)} \quad \text{and} \quad \Delta_T = \frac{1}{2g} \left[ \frac{1-2g}{1-g} \epsilon^{(V)} - \delta^{(V)} \right]. \quad (6)$$

Both weaknesses can be found from  $P$ -wave traveltime data acquired in wide-azimuth surveys if an estimate of  $V_S/V_P$  ratio is known. Indeed, the  $P$ -wave NMO ellipse in a plane homogeneous HTI layer can be written as (Tsvankin, 1997)

$$V_{P,\text{nmo}}^2(\beta) = V_{P0}^2 \frac{1 + 2\delta^{(V)}}{1 + 2\delta^{(V)} \sin^2 \beta}, \quad (7)$$

where  $V_{P0}$  is the  $P$ -wave vertical velocity, and  $\beta$  is the azimuth of the CMP line with respect to the symmetry axis (normal to the fractures). Therefore,  $P$ -wave NMO velocity measurements along several azimuths (three are generally sufficient) can be inverted for the fracture orientation,  $V_{P0}$  and  $\delta^{(V)}$ .

The value of  $\epsilon^{(V)}$  (or  $\eta^{(V)}$ ) can be found using NMO velocities from dipping reflectors (Tsvankin, 1997; Contreras et al., 1999) or nonhyperbolic moveout. If no such data are available, the inversion can be performed by combining the  $P$ -wave NMO ellipse from horizontal reflectors with the azimuthally varying  $P$ -wave AVO gradient that depends on  $\delta^{(V)}$  and the splitting parameter  $\gamma^{(V)}$ .

Next, we assume that  $\epsilon^{(V)}$  and  $\delta^{(V)}$  have already been estimated and examine whether or not it is possible to distinguish between dry and fluid-filled cracks in the presence of realistic errors in  $\epsilon^{(V)}$ ,  $\delta^{(V)}$ , and the  $V_S/V_P$  ratio. First, consider dry and fluid-filled penny-shaped cracks with the crack density  $e = 7\%$  and  $V_S/V_P = 0.5$ . For dry cracks,  $\Delta_N = 0.50$ ,  $\epsilon^{(V)} = -0.21$ , and  $\delta^{(V)} = -0.19$ ; for fluid-filled cracks,  $\Delta_N = 0$ ,  $\epsilon^{(V)} = 0$ , and  $\delta^{(V)} = -0.07$  ( $\Delta_T = 0.15$  in both cases). We simulated errors in the data by adding Gaussian noise with the standard deviation  $\sigma = 0.05$  to the correct values of  $\epsilon^{(V)}$ ,  $\delta^{(V)}$ , and  $V_S/V_P$  and carried out the inversion for  $\Delta_N$  and  $\Delta_T$  using the exact equations. The inversion results, along with the input anisotropic coefficients, are marked by small dots in Figure 2. Clearly, the clouds of the estimated  $\Delta_N$  and  $\Delta_T$  values are well separated, which indicates the possibility of resolving the fracture content for errors in  $\epsilon^{(V)}$  and  $\delta^{(V)}$  on the order of  $\pm 0.05 - 0.1$ .

### Conclusions

The linear slip theory, based on the general treatment of fractures as surfaces of weakness inside a rock, provides a convenient framework for relating seismic signatures to the properties of fracture systems. The inherent parameters of the linear slip theory (Schoenberg and Sayers, 1995) for rotationally invariant fractures are the normal  $\Delta_N$  and tangential  $\Delta_T$  weaknesses that can be

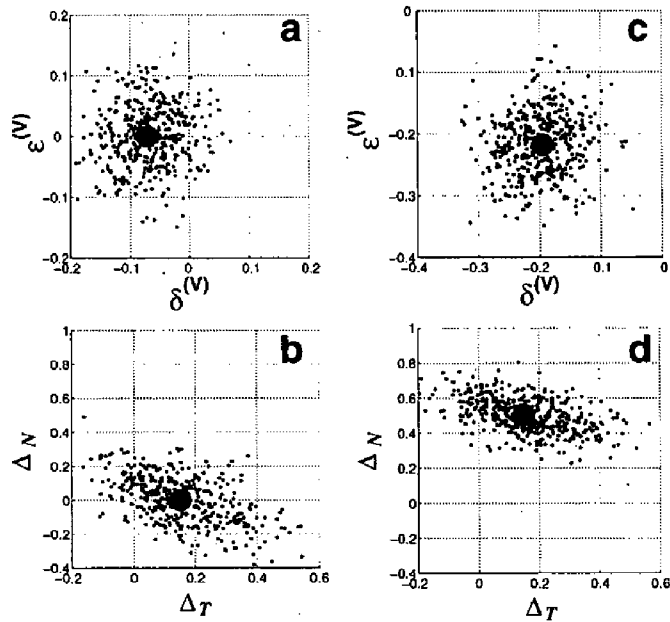


FIG. 2. Weaknesses  $\Delta_N$  and  $\Delta_T$  (small dots) for fluid-filled (b) and dry (d) cracks obtained by inverting the anisotropic coefficients  $\epsilon^{(V)}$  and  $\delta^{(V)}$  shown in (a) and (c), respectively. The large dots indicate the correct values of all parameters.

estimated from seismic data. Theories of Hudson (1981) and Thomsen (1995), which operate with specific physical fracture models, can be used to express  $\Delta_N$  and  $\Delta_T$  through the parameters dependent on the microstructure of cracks and pores. Although it may be difficult to obtain these additional parameters unambiguously from seismic data, Hudson's and Thomsen's models are helpful in guiding the interpretation of the weaknesses in terms of the crack density and fluid content.

Vertical, parallel, rotationally invariant cracks lead to a particular type of transversely isotropic media with a horizontal symmetry axis (HTI) described by four independent parameters. We obtained simple linearized expressions for the Thomsen-style anisotropic coefficients  $\epsilon^{(V)}$ ,  $\delta^{(V)}$ , and  $\gamma^{(V)}$  in terms of  $\Delta_N$ ,  $\Delta_T$  and the  $V_S/V_P$  ratio in the background medium. Hudson's theory was used to study the dependence of the weaknesses and anisotropic parameters on the physical properties of penny-shaped cracks. The parameters  $\epsilon^{(V)}$  and  $\delta^{(V)}$  can be obtained either from azimuthally dependent  $P$ -wave traveltime data alone (using dipping events or nonhyperbolic moveout) or by combining the  $P$ -wave NMO ellipse from horizontal reflectors with the azimuthal variation of the AVO gradient. Then  $\epsilon^{(V)}$  and  $\delta^{(V)}$ , along with an estimate of the  $V_S/V_P$  ratio, can be inverted for the weaknesses  $\Delta_N$  and  $\Delta_T$  which provide information about the crack density and fluid saturation (i.e., allow one to distinguish between dry and fluid-filled cracks).

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