2-01 ELASTIC WAVES IN THREE-PHASE POROELASTIC MEDIA

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Introduction

Real rocks are often multiphase. Two-phase porous rocks consisting of skeleton and fluid are well described by Biot model. Due to polycrystalline nature of rocks skeleton is also may be inhomogeneous. Different solid phases or grains may be non-perfectly welded or even may slide along each other [1]. If this is the case, wave propagation becomes significantly different from the case of monophase elastic rock or even two-phase rock [2]. This behavior is important for understanding nature of propagating waves, their velocities and amplitudes in heterogeneous rocks. Presence of additional waves significantly changes the energy balance and hence amplitude of conventional reflected P-waves. Although general case of even three-phase media is too complicated, some physical insight into wave propagation may be gained while considering simplified finely layered models.

Equations of three-phase model with two solid and one fluid phase

Here we study wave propagation in finely layered media consisting of two different alternating porous layers. If welded contact is assumed between skeleton of two layers then medium is generalization of two-phase Biot model [3]. When total slip is assumed along interfaces ($\tau_{xz} = 0$) then effective (long-wave equivalent media) model is three-phase [4]. Physically it happens because two different solid phases may now independently move along lamination. Hence this model has two different solid phases and one fluid phase. For rocks it may be, for example, ice, mineral grains and water in frozen rocks [1] because ice may be imperfectly welded with mineral grains. We denote displacement and stresses of two different solid phases by subscript i ($i = 1, 2$). We also assume that porous layers of first type occupies θ_1 relative portion of total volume. Likewise, second type occupies relative portion θ_2 ($\theta_1 + \theta_2 = 1$). Constitutive equations of such three-phase finely layered media are given by

$$\begin{aligned}
\theta_1 \tau_{xx1} &= P_{11}\frac{\partial u_{x1}}{\partial x} + P_{12}\frac{\partial u_{x2}}{\partial x} + F_1\frac{\partial u_z}{\partial z} + M_1\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}\right),\\
\theta_2 \tau_{xx2} &= P_{12}\frac{\partial u_{x1}}{\partial x} + P_{22}\frac{\partial u_{x2}}{\partial x} + F_2\frac{\partial u_z}{\partial z} + M_2\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}\right),\\
\tau_{zz} &= F_1\frac{\partial u_{x1}}{\partial x} + F_2\frac{\partial u_{x2}}{\partial x} + C\frac{\partial u_z}{\partial z} + Q\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}\right),\\
-p &= M_1\frac{\partial u_{x1}}{\partial x} + M_2\frac{\partial u_{x2}}{\partial x} + Q\frac{\partial u_z}{\partial z} + R\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z}\right),
\end{aligned} \tag{1}$$

$$\theta_1\frac{\partial \tau_{xx1}}{\partial x} = \rho_{x11}\ddot{u}_{x1} + \rho_{x12}\ddot{u}_{x2} + \rho_{fx1}\ddot{w}_x,\ \theta_2\frac{\partial \tau_{xx2}}{\partial x} = \rho_{x12}\ddot{u}_{x1} + \rho_{x22}\ddot{u}_{x2} + \rho_{fx2}\ddot{w}_x,$$

$$\frac{\partial \tau_{zz}}{\partial z} = \rho_z\ddot{u}_z + \rho_{fz}\ddot{w}_z,\ -\frac{\partial p}{\partial z} = \rho_{fz}\ddot{u}_z + m_z\ddot{w}_z, -\frac{\partial p}{\partial x} = \rho_{fx1}\ddot{u}_{x1} + \rho_{fx2}\ddot{u}_{x2} + m_x\ddot{w}_x, \tag{2}$$

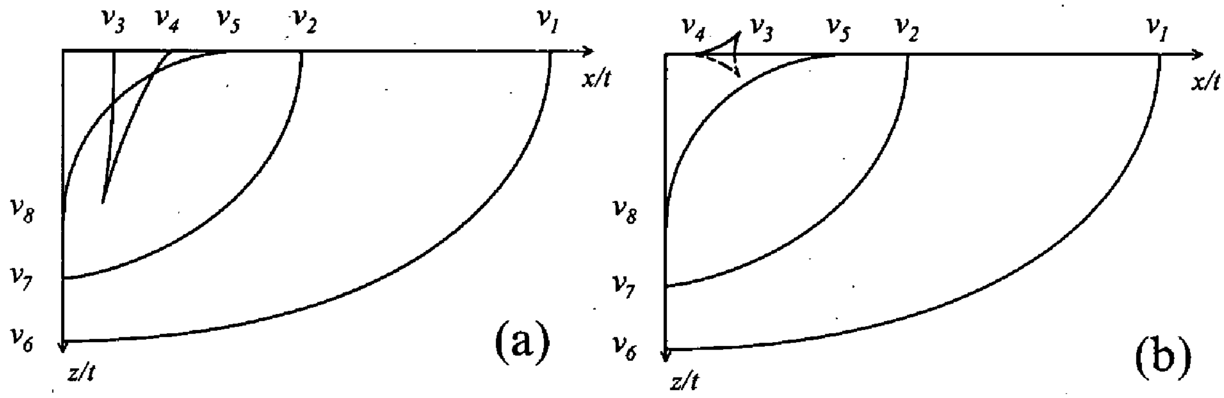

Fig. 1. Group velocities in three-phase poroelastic model of alternating porous layers with slip boundary conditions. (a) and (b) show different shapes of "triangular" wave front of third longitudinal wave which is due to the relative motion of two solid phases.

where u_{xi} is displacement of i-th solid phase, τ_{xxi} is total stress of fluid and solid phase i $(i = 1, 2)$, u_z, τ_{zz} are common for all three phases displacement and stress across lamination, w_x, w_z, p are displacement of fluid relative to solid phase and fluid pressure. Coefficients in equations (1) correspond to different elastic moduli whereas coefficients in (2) correspond to different densities. Shear stress τ_{xz} is assumed to be zero everywhere due to boundary conditions (total slip).

Discussion of wave propagation

Christoffel equation for three-phase model (in 2D case) is of 8th order. It means that 4 waves propagate in the model. Typical group velocity curves (wave fronts from a point source) are shown on Figure 1a and 1b. Identifying the nature of those waves it is easy to show that fast wave (with velocities v_1 and v_6) corresponds to conventional first longitudinal wave where displacements in solids and fluid are in phase. Second wave front (with velocities v_2 and v_7) corresponds to so called "slow" or second longitudinal wave typical for two phase poroelastic Biot model. This wave is due to the relative motion of fluid and solids. These displacements are out of phase in this wave. Third "triangular" wave front (with velocities v_3 and v_4) is also of longitudinal character. This wave propagates because in three-phase model the relative motion of two solid phases is also possible. This type of relative motion is allowed only along lamination and is forbidden across (u_z is equal in all three phases due to boundary conditions). Hence velocity of this "third" longitudinal wave becomes zero in vertical direction, thus giving "triangular" shape to the wave front. Forth concave wave front (with velocities v_5 and v_8) is of quasi-shear character and is formed as remainder of the loop when velocities along axes become zero ($\tau_{xz} = 0$).

References

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