

SOURCES OF THE CENTER OF DILATATION AND CENTER OF COMPRESSION TYPES IN THE BIOT MODEL

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For a homogeneous isotropic model of porous Biot media, wave fields of spherically symmetric point sources are determined. The conditions under which a point source of the center of compression type can be replaced by two sources, one of which is a pair of oppositely directed forces and the other is a center of radially directed tangential forces, are obtained. Bibliography: 9 titles.

In this paper, the wave fields of point sources of the center of dilatation and center of compression types are determined for the homogeneous isotropic unbounded Biot model of a porous medium. Each of these sources is initially given on a sphere of small radius R_0 . The interior of this sphere can be either empty or filled with the same medium. In the case of an empty ball $R < R_0$, stresses and displacements on the sphere $R = R_0$ are given. Such a source is referred to as a source of the first kind [1]. In the other case, the jumps of stresses and displacements are given on the sphere $R = R_0$, which no longer coincides with the interface between the two media. Such a source is said to be of the second kind [1].

For the above spherical sources, we determine the wave field, in which passage to the limit is carried out as $R_0 \rightarrow 0$. In passing from spherical sources to point ones, the conditions of finiteness of the wave fields are found. These conditions characterize the increase of displacements and stresses or of their jumps on the sphere $R = R_0$ as $R_0 \rightarrow 0$. In addition, for sources of the center of compression type of the second kind, the conditions under which the Saint-Venant principle is applicable to the Biot model are determined. In those particular cases where the Biot model converts into an elastic or a fluid medium, the expressions for the wave field turn into the corresponding expressions for elastic and fluid media [1]. Such a correspondence is established for sources of the second kind.

Fields of sources in porous media described by the Biot model were considered in a number of papers [2–6]. In most papers [2–5], a source of the type of directed forces, which are taken into account in the right-hand sides of wave equations, in an unbounded homogeneous isotropic porous medium was considered. This approach is complicated by the fact that only four among the six components of forces are independent. In [4], in order that the system of equations be neither indefinite nor overdetermined, three components of displacements in the frame and pressure in the fluid phase are used as independent variables, and the Biot equations are replaced by four equations analogous to the equations of thermoelasticity. The method used in this paper is free from the above-indicated complications.

§1. SOURCES OF THE FIRST KIND

In a spherical system of coordinates R, θ, z , consider a homogeneous isotropic porous Biot medium with a spherical excision $R < R_0$. On the boundary $R = R_0$ of this excision a certain spherical symmetric source is given by the relations

$$\sigma^{(1)} = B_1 f_1(t), \quad \sigma_{RR}^{(2)} = B_2 f_2(t) \quad (1.1)$$

or

$$u_R^{(1)} = B_3 f_3(t), \quad u_R^{(2)} = B_4 f_4(t), \quad (1.2)$$

where $R = R_0$. This source acts since the time $t = 0$. The variables $u_R^{(1)}$, $u_R^{(2)}$, $\sigma^{(1)}$, and $\sigma_{RR}^{(2)}$ describe certain displacements and stresses, while the superscripts 1 and 2 are related to the fluid and elastic phases, respectively. As in [7], the averaging of stresses is carried out upon the total volume of a small neighborhood of the point considered, but the displacements are averaged in this neighborhood separately

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upon the volumes of each of the phases. The wave field excited is expressed in terms of the potentials φ_1 and φ_2 by the equalities

$$\begin{aligned} u_R^{(1)} &= \frac{\partial \varphi_1}{\partial R}, \quad u_R^{(2)} = \frac{\partial \varphi_2}{\partial R}, \\ \sigma^{(1)} &= \tilde{R} \left(\frac{\partial^2 \varphi_1}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_1}{\partial R} \right) + M \left(\frac{\partial^2 \varphi_2}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_2}{\partial R} \right), \\ \sigma_{RR}^{(2)} &= M \left(\frac{\partial^2 \varphi_1}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_1}{\partial R} \right) + P \frac{\partial^2 \varphi_2}{\partial R^2} + \frac{2F}{R} \frac{\partial \varphi_2}{\partial R}. \end{aligned} \quad (1.3)$$

The potentials satisfy the equations

$$\begin{aligned} P \left(\frac{\partial^2 \varphi_2}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_2}{\partial R} \right) + M \left(\frac{\partial^2 \varphi_1}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_1}{\partial R} \right) &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ M \left(\frac{\partial^2 \varphi_2}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_2}{\partial R} \right) + \tilde{R} \left(\frac{\partial^2 \varphi_1}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi_1}{\partial R} \right) &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}. \end{aligned} \quad (1.4)$$

In relations (1.3) and (1.4), the notation suggested by Biot [8] is used. The coefficients $P, F, M, \tilde{R}, \rho_{11}, \rho_{12}$, and ρ_{22} satisfy the inequalities

$$P\tilde{R} - M^2 > 0, \quad P > F, \quad P > 0, \quad \tilde{R} > 0, \quad \rho_{11}\rho_{22} - \rho_{12}^2 > 0, \quad (1.5)$$

insuring the positivity of the densities of kinetic and potential energies.

In considering spherically symmetric wave fields, it is useful to pass to other potentials ϕ_1 and ϕ_2 by using the relations

$$\varphi_1 = \phi_1/R, \quad \varphi_2 = \phi_2/R. \quad (1.6)$$

The new potentials satisfy the following equations:

$$\begin{aligned} P \frac{\partial^2 \phi_2}{\partial R^2} + M \frac{\partial^2 \phi_1}{\partial R^2} &= \rho_{11} \frac{\partial^2 \phi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \phi_1}{\partial t^2}, \\ M \frac{\partial^2 \phi_2}{\partial R^2} + \tilde{R} \frac{\partial^2 \phi_1}{\partial R^2} &= \rho_{12} \frac{\partial^2 \phi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \phi_1}{\partial t^2}. \end{aligned} \quad (1.7)$$

In terms of the new potentials, displacements and stresses are expressed as

$$\begin{aligned} u_R^{(1)} &= \frac{1}{R} \frac{\partial \phi_1}{\partial R} - \frac{\phi_1}{R^2}, \quad u_R^{(2)} = \frac{1}{R} \frac{\partial \phi_2}{\partial R} - \frac{\phi_2}{R^2}, \\ \sigma^{(1)} &= \frac{1}{R} \left(\rho_{12} \frac{\partial^2 \phi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \phi_1}{\partial t^2} \right), \\ \sigma_r^{(2)} &= \frac{1}{R} \left(\rho_{11} \frac{\partial^2 \phi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \phi_1}{\partial t^2} \right) - \frac{4L}{R^2} \left(\frac{\partial \phi_2}{\partial R} - \frac{\phi_2}{R} \right), \end{aligned} \quad (1.8)$$

where $L = (P - F)/2$.

We represent the solution of the system of equations (1.7) with constant coefficients in the form

$$\begin{aligned} \phi_1 &= F_1 \left(t - \frac{R}{V_1} \right) + F_2 \left(t - \frac{R}{V_2} \right), \\ \phi_2 &= A_1 F_1 \left(t - \frac{R}{V_1} \right) + A_2 F_2 \left(t - \frac{R}{V_2} \right), \end{aligned} \quad (1.9)$$

where F_1 and F_2 are arbitrary functions in their arguments. The equalities

$$V_{1,2}^2 = \left[\pm \sqrt{(P\rho_{22} + \tilde{R}\rho_{11} - 2M\rho_{12})^2 - 4(\rho_{11}\rho_{12} - \rho_{12}^2)(P\tilde{R} - M^2)} + \right. \\ \left. + P\rho_{22} + \tilde{R}\rho_{11} - 2M\rho_{12} \right] / [2(\rho_{11}\rho_{22} - \rho_{12}^2)], \quad (1.10)$$

$$A_{1,2} = \frac{\rho_{12}V_{1,2}^2 - M}{P - \rho_{11}V_{1,2}^2} = \frac{\rho_{22}V_{1,2}^2 - \tilde{R}}{M - \rho_{12}V_{1,2}^2} \quad (1.11)$$

for the velocities of propagation V_1, V_2 and the relative amplitudes A_1 and A_2 follow from the compatibility conditions for the solutions (1.9) of Eqs. (1.7). Moreover, in view of inequalities (1.5), the inequalities $V_1 > 0$ and $V_2 > 0$ are valid. On substituting formulas (1.9) into relations (1.8), we arrive at the equalities

$$u_R^{(1)} = -\frac{1}{RV_1}F_1' \left(t - \frac{R}{V_1} \right) - \frac{1}{RV_2}F_2' \left(t - \frac{R}{V_2} \right) - \frac{1}{R^2}F_1 \left(t - \frac{R}{V_1} \right) - \frac{1}{R^2}F_2 \left(t - \frac{R}{V_2} \right), \\ u_R^{(2)} = -\frac{A_1}{RV_1}F_1' \left(t - \frac{R}{V_1} \right) - \frac{A_2}{RV_2}F_2' \left(t - \frac{R}{V_2} \right) - \frac{A_1}{R^2}F_1 \left(t - \frac{R}{V_1} \right) - \frac{A_2}{R^2}F_2 \left(t - \frac{R}{V_2} \right), \\ \sigma_{RR}^{(2)} = \frac{1}{R} \left[(\rho_{12} + \rho_{11}A_1)F_1'' \left(t - \frac{R}{V_1} \right) + (\rho_{12} + \rho_{11}A_2)F_2'' \left(t - \frac{R}{V_2} \right) \right] - \frac{4L}{R}u_R^{(2)}, \\ \sigma^{(1)} = \frac{1}{R} \left[(\rho_{11} + \rho_{12}A_1)F_1'' \left(t - \frac{R}{V_1} \right) + (\rho_{11} + \rho_{12}A_2)F_2'' \left(t - \frac{R}{V_2} \right) \right]. \quad (1.12)$$

If we take into account the third and fourth relations in (1.12), boundary conditions (1.1), defining a source of the center of compression type, are replaced by the equalities

$$R_0^2 \left[(\rho_{12} + \rho_{11}A_1)F_1'' \left(t - \frac{R_0}{V_1} \right) + (\rho_{12} + \rho_{11}A_2)F_2'' \left(t - \frac{R_0}{V_2} \right) \right] + \\ + 4LR_0 \left[\frac{A_1}{V_1}F_1' \left(t - \frac{R_0}{V_1} \right) + \frac{A_2}{V_2}F_2' \left(t - \frac{R_0}{V_2} \right) \right] + \\ + 4L \left[A_1F_1 \left(t - \frac{R_0}{V_1} \right) + A_2F_2 \left(t - \frac{R_0}{V_1} \right) \right] = B_2R_0^3f_2(t), \\ (\rho_{11} + \rho_{12}A_1)F_1'' \left(t - \frac{R_0}{V_1} \right) + (\rho_{11} + \rho_{12}A_2)F_2'' \left(t - \frac{R_0}{V_2} \right) = B_1R_0f_1(t). \quad (1.13)$$

To pass in formulas (1.13) to the point source of the center of compression type, it is necessary to pass to the limit as $R_0 \rightarrow 0$ and to require that the quantities B_1R_0 and $B_2R_0^3$ remain finite. Similar conditions were imposed in [1] when considering point sources of the center of compression type in elastic and fluid media. Since in practice it is apparently impossible to satisfy both of these requirements simultaneously, we set

$$B_1R_0 = 1, \quad B_2R_0^3 = 0 \quad (1.14)$$

and arrive at the equalities

$$A_1F_1(t) + A_2F_2(t) = 0, \\ (\rho_{11} + \rho_{12}A_1)F_1''(t) + (\rho_{11} + \rho_{12}A_2)F_2''(t) = f_1(t), \quad (1.15)$$

which show that the Biot medium under consideration is excited by the point source only through its fluid phase. After differentiating the first equality in (1.15) twice and eliminating one of the second derivatives, we obtain the expressions

$$F_1''(t) = \frac{A_2f_1(t)}{\rho_{11}(A_2 - A_1)}, \quad F_2''(t) = -\frac{A_1f_1(t)}{\rho_{11}(A_2 - A_1)}. \quad (1.16)$$

It is convenient to also represent the wave field obtained in the cylindrical system of coordinates r, φ, z . If we represent a function $f(t - R/v)$, defined only for positive values of its argument, by the Mellin integral

$$f\left(t - \frac{R}{v}\right) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} G(s) e^{s(t-\frac{R}{v})} ds, \quad (1.17)$$

use the formula [9]

$$\frac{e^{-SR/v}}{R} = \int_0^\infty k J_0(kr) \frac{e^{-|z|\sqrt{k^2+s^2/v^2}}}{\sqrt{k^2+s^2/v^2}} dk, \quad (1.18)$$

permute the integrals, and change the variable of integration in accordance with the formula $s = k\eta$, then we obtain the relation

$$f(t - R/v)/R = \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{G(k\eta) e^{\kappa(t\eta-|z|\alpha)}}{\alpha} d\eta, \quad (1.19)$$

where

$$\alpha = \sqrt{1 + \eta^2/v^2}, \quad G(k\eta) = \int_0^\infty f(t) e^{-k\eta t} dt. \quad (1.20)$$

Using formula (1.19), for the wave field (1.6), (1.9), (1.16) in the cylindrical coordinates we obtain the representations

$$\varphi_i = \int_0^\infty \frac{J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Z_i e^{k\eta t} d\eta, \quad (1.21)$$

where

$$Z_1 = \frac{G_1(k\eta)}{\rho_{11}(A_2 - A_1)k^2\eta^2} \left(\frac{A_2 e^{-k|z|\alpha_1}}{\alpha_1} - \frac{A_1 e^{-k|z|\alpha_2}}{\alpha_2} \right), \quad (1.22)$$

$$Z_2 = -\frac{G_1(k\eta)A_1A_2}{\rho_{11}(A_2 - A_1)k^2\eta^2} \left(\frac{e^{-k|z|\alpha_1}}{\alpha_1} - \frac{e^{-k|z|\alpha_2}}{\alpha_2} \right);$$

$$\alpha_i = \sqrt{1 + \eta^2/v_i^2} \quad (i = 1, 2), \quad G_1(k\eta) = \int_0^\infty f_1(t) e^{-k\eta t} dt. \quad (1.23)$$

Now we consider the action of a source of the center of dilatation type. In this case, the problem of finding the field reduces to the determination of the functions $F_1(t)$ and $F_2(t)$ from the system of equations

$$\begin{aligned} \frac{R_0}{V_1} F_1' \left(t - \frac{R_0}{V_1} \right) + F_1 \left(t - \frac{R_0}{V_1} \right) + \frac{R_0}{V_2} F_2' \left(t - \frac{R_0}{V_2} \right) + F_2 \left(t - \frac{R_0}{V_2} \right) &= -B_1 R_0^2 f_1(t), \\ A_1 \left[\frac{R_0}{V_1} F_1' \left(t - \frac{R_0}{V_1} \right) + F_1 \left(t - \frac{R_0}{V_1} \right) \right] + A_2 \left[\frac{R_0}{V_2} F_2' \left(t - \frac{R_0}{V_2} \right) + F_2 \left(t - \frac{R_0}{V_2} \right) \right] &= -B_2 R_0^2 f_2(t), \end{aligned} \quad (1.24)$$

which is based on the first and second relations in (1.12). In equalities (1.24), we carry out limit passage as $R_0 \rightarrow 0$, and, in order to obtain finite quantities, we assume that the coefficients B_1 and B_2 satisfy the conditions

$$B_1 R_0^2 = \bar{B}_1 = \text{const}, \quad B_2 R_0^2 = \bar{B}_2 = \text{const}. \quad (1.25)$$

After this passage to the limit system (1.24) is replaced by the equalities

$$F_1(t) + F_2(t) = -\bar{B}_1 f_1(t), \quad A_1 F_1(t) + A_2 F_2(t) = -\bar{B}_2 f_2(t), \quad (1.26)$$

from which the expressions

$$F_1(t) = -\frac{A_2 \bar{B}_1 f_1(t) - \bar{B}_2 f_2(t)}{A_2 - A_1}, \quad F_2(t) = \frac{\bar{B}_1 A_1 f_1(t) - \bar{B}_2 f_2(t)}{A_2 - A_1} \quad (1.27)$$

for the functions $F_1(t)$ and $F_2(t)$ follow. Passage to the cylindrical coordinates in the wave field constructed is performed by using formula (1.18). As previously, the potentials are represented by relations (1.21), where now

$$Z_1 = \frac{k}{A_2 - A_1} \left[(\bar{B}_2 G_2 - A_2 \bar{B}_1 G_1) \frac{e^{-k|z|\alpha_1}}{\alpha_1} + (\bar{B}_1 A_1 G_1 - \bar{B}_2 G_2) \frac{e^{-k|z|\alpha_2}}{\alpha_2} \right],$$

$$Z_2 = \frac{k}{A_2 - A_1} \left[A_1 (\bar{B}_2 G_2 - A_2 \bar{B}_1 G_1) \frac{e^{-k|z|\alpha_1}}{\alpha_1} + A_2 (\bar{B}_1 A_1 G_1 - \bar{B}_2 G_2) \frac{e^{-k|z|\alpha_2}}{\alpha_2} \right], \quad (1.28)$$

$$G_i = G_i(k\eta) = \int_0^\infty f_i(t) e^{-k\eta t} dt \quad (i = 1, 2).$$

Thus, formulas (1.21) and (1.28) determine the wave field excited by a source of the center of dilatation type in the cylindrical system of coordinates.

§2. SOURCES OF THE SECOND KIND

In the spherical system of coordinates R, θ, φ , consider a homogeneous isotropic unbounded porous Biot medium excited by a source applied to the sphere $R = R_0$ and acting since $t = 0$. The source is given by the boundary conditions

$$[\sigma^{(1)}] = B_1 f_1(t), \quad [\sigma_{RR}^{(2)}] = B_2 f_2(t),$$

$$[u_R^{(1)}] = B_3 f_3(t), \quad [u_R^{(2)}] = B_4 f_4(t) \quad (2.1)$$

for $R = R_0$ and is called, according to [1], a source of the second kind. The symbol $[]$, occurring in equalities (2.1), denotes the jump at $R = R_0$ of the function in square brackets.

The field excited satisfies relations (1.3)–(1.8) and is characterized by the potentials $\varphi_1 = \phi_1/R$ and $\varphi_2 = \phi_2/R$. The quantities ϕ_1 and ϕ_2 are solutions of Eqs. (1.7) and can be represented as follows:

$$\phi_1 = \begin{cases} F_1 \left(t - \frac{R-R_0}{V_1} \right) + F_2 \left(t - \frac{R-R_0}{V_2} \right), & R > R_0, \\ F_{10} \left(t + \frac{R+R_0}{V_1} \right) - F_{10} \left(t - \frac{R-R_0}{V_1} \right) + F_{20} \left(t + \frac{R+R_0}{V_2} \right) - F_{20} \left(t - \frac{R-R_0}{V_2} \right), & R < R_0; \end{cases} \quad (2.2)$$

$$\phi_2 = \begin{cases} A_1 F_1 \left(t - \frac{R-R_0}{V_1} \right) + A_2 F_2 \left(t - \frac{R-R_0}{V_2} \right), & R > R_0, \\ A_1 F_{10} \left(t + \frac{R+R_0}{V_1} \right) - A_1 F_{10} \left(t - \frac{R-R_0}{V_1} \right) + A_2 F_{20} \left(t + \frac{R+R_0}{V_2} \right) - A_2 F_{20} \left(t - \frac{R-R_0}{V_2} \right), & R < R_0, \end{cases}$$

where F_1, F_2, F_{10} , and F_{20} are arbitrary functions in their arguments. In the region $R > R_0$, these functions describe waves going to infinity. However, inside the sphere $R = R_0$ they describe waves propagating both from the center and toward it; in addition, the conditions $\phi_1(0) = \phi_2(0) = 0$, insuring the finiteness of the potentials φ_1 and φ_2 at the center of the sphere, are fulfilled.

The determination of the field excited is thus reduced to the determination of explicit expressions for the functions $F_1(t)$, $F_2(t)$, $F_{10}(t)$, and $F_{20}(t)$ from boundary conditions (2.1). To this end, using (1.8), we represent the left-hand sides of equalities (2.1) by the relations

$$[u_R^{(1)}] = \frac{1}{R_0} \left[\frac{\partial \phi_1}{\partial R} \right] - \frac{1}{R_0^2} [\phi_1],$$

$$[u_R^{(2)}] = \frac{1}{R_0} \left[\frac{\partial \phi_2}{\partial R} \right] - \frac{1}{R_0^2} [\phi_2],$$

$$[\sigma^{(1)}] = \frac{1}{R_0} \frac{\partial^2}{\partial t^2} (\rho_{12} [\phi_2] + \rho_{22} [\phi_1]),$$

$$[\sigma_{RR}^{(2)}] = \frac{1}{R_0} \frac{\partial^2}{\partial t^2} (\rho_{11} [\phi_2] + \rho_{12} [\phi_1]) - \frac{4L}{R_0} [u_R^{(2)}]. \quad (2.3)$$

The jumps of the potentials ϕ_1 and ϕ_2 and of their derivatives are expressed as follows:

$$\begin{aligned}
 [\phi_1] &= F_1(t) + F_2(t) + F_{10} \left(t - \frac{2R_0}{V_1} \right) - F_{10}(t) + F_{20} \left(t - \frac{2R_0}{V_2} \right) - F_{20}(t), \\
 [\phi_2] &= A_1 F_1(t) + A_2 F_2(t) + A_1 F_{10} \left(t - \frac{2R_0}{V_1} \right) - A_1 F_{10}(t) + A_2 F_{20} \left(t - \frac{2R_0}{V_2} \right) - A_2 F_{20}(t), \\
 \left[\frac{\partial \phi_1}{\partial R} \right] &= -\frac{1}{V_1} F'_1(t) - \frac{1}{V_2} F'_2(t) - \frac{1}{V_1} F'_{10} \left(t - \frac{2R_0}{V_1} \right) - \frac{1}{V_1} F'_{10}(t) - \frac{1}{V_2} F'_{20} \left(t - \frac{2R_0}{V_2} \right) - \frac{1}{V_2} F'_{20}(t), \\
 \left[\frac{\partial \phi_2}{\partial R} \right] &= -\frac{A_1}{V_1} F'_1(t) - \frac{A_2}{V_2} F'_2(t) - \frac{A_1}{V_1} F'_{10} \left(t - \frac{2R_0}{V_1} \right) - \frac{A_1}{V_1} F'_{10}(t) - \frac{A_2}{V_2} F'_{20} \left(t - \frac{2R_0}{V_2} \right) - \frac{A_2}{V_2} F'_{20}(t).
 \end{aligned} \tag{2.4}$$

On the basis of relations (2.1), (2.3), and (2.4), the system of four linear operator equations in the functions F_1 , F_2 , F_{10} , and F_{20} can be written. This system can be solved, in particular, by applying the Laplace transform, and its solution determines the wave field of the spherical source. Passage to the limit as $R_0 \rightarrow 0$ in the expressions for F_1 , F_2 , F_{10} , and F_{20} yields formulas for the field of a point source.

However, for simplicity of transformations, we confine ourselves to substituting formulas (2.4) into relations (2.3). In the expression obtained we use expansions in Taylor series and retain, for a small R_0 , only the highest terms in the coefficients at the functions $F_1(t)$, $F_2(t)$, $F_{10}(t)$, and $F_{20}(t)$ or at their derivatives. As a result we obtain the representations

$$\begin{aligned}
 [u_R^{(1)}] &= -\frac{1}{R_0^2} F_1(t) - \frac{1}{R_0^2} F_2(t) - \frac{2}{3} \frac{R_0}{V_1^3} F'''_{10}(t) - \frac{2}{3} \frac{R_0}{V_2^3} F'''_{20}(t), \\
 [u_R^{(2)}] &= -\frac{A_1}{R_0^2} F_1(t) - \frac{A_2}{R_0^2} F_2(t) - \frac{2}{3} \frac{A_1 R_0}{V_1^3} F'''_{10}(t) - \frac{2}{3} \frac{A_2 R_0}{V_2^3} F'''_{20}(t), \\
 R_0[\sigma^{(1)}] &= (\rho_{12} A_1 + \rho_{22}) F''_1(t) + (\rho_{12} A_2 + \rho_{22}) F''_2(t) - \\
 &\quad - \frac{2R_0}{V_1} (\rho_{12} A_1 + \rho_{22}) F'''_{10}(t) - \frac{2R_0}{V_2} (\rho_{12} A_2 + \rho_{22}) F'''_{20}(t), \\
 R_0[\sigma_{RR}^{(2)}] + 4L[u_R^{(2)}] &= (\rho_{11} A_1 + \rho_{12}) F''_1(t) + (\rho_{11} A_2 + \rho_{12}) F''_2(t) - \\
 &\quad - \frac{2R_0}{V_1} (\rho_{11} A_1 + \rho_{12}) F'''_{10}(t) - \frac{2R_0}{V_2} (\rho_{11} A_2 + \rho_{12}) F'''_{20}(t).
 \end{aligned} \tag{2.5}$$

Then from the first two equations in (2.5) we find expressions for $F'''_{10}(t)$ and $F'''_{20}(t)$, substitute them into the third and fourth equations, and in the equalities obtained ignore the terms involving $F''_1(t)$ and $F''_2(t)$, which are negligible for a small R_0 . These operations result in the following expressions for the functions $F_1(t)$ and $F_2(t)$:

$$\begin{aligned}
 F_1(t) &= -\frac{R_0^3}{3V_1^2(A_1 - A_2)(\rho_{11}\rho_{22} - \rho_{12}^2)} \left[(\rho_{11} A_2 + \rho_{12}) [\sigma^{(1)}] - (\rho_{12} A_2 + \rho_{22}) [\sigma_{RR}^{(2)}] \right] + \\
 &\quad + \frac{R_0^2(A_2[u_R^{(1)}] - [u_R^{(2)}])}{A_1 - A_2} + \frac{4L(\rho_{12} A_2 + \rho_{22}) R_0^2 [u_R^{(2)}]}{3V_1^2(A_1 - A_2)(\rho_{11}\rho_{22} - \rho_{12}^2)}, \\
 F_2(t) &= \frac{R_0^3[(\rho_{11} A_1 + \rho_{12}) [\sigma^{(1)}] - (\rho_{12} A_1 + \rho_{22}) [\sigma_{RR}^{(2)}]]}{3V_2^2(A_1 - A_2)(\rho_{11}\rho_{22} - \rho_{12}^2)} - \\
 &\quad - \frac{R_0^2(A_1[u_R^{(1)}] - [u_R^{(2)}])}{A_1 - A_2} - \frac{4L(\rho_{12} A_1 + \rho_{22}) R_0^2 [u_R^{(2)}]}{3V_2^2(A_1 - A_2)(\rho_{11}\rho_{22} - \rho_{12}^2)}.
 \end{aligned} \tag{2.6}$$

In order that the functions $F_1(t)$ and $F_2(t)$ be finite as $R_0 \rightarrow 0$, it is necessary that the conditions

$$\begin{aligned}
 B_1 R_0^3 = \tilde{B}_1 = \text{const}, \quad B_2 R_0^3 = \tilde{B}_2 = \text{const}, \\
 B_3 R_0^2 = \tilde{B}_3 = \text{const}, \quad B_4 R_0^2 = \tilde{B}_4 = \text{const}
 \end{aligned} \tag{2.7}$$

be satisfied. Relations (2.1) and (2.7) must be taken into account in introducing corrections into formulas (2.6).

It is useful to split the source of the second kind, given by equalities (2.10), into two sources one of which is of the center of compression type,

$$[\sigma^{(1)}] = \tilde{B}_1 f_1(t)/R_0^3, \quad [\sigma_{RR}^{(2)}] = \tilde{B}_2 f_2(t)/R_0^3, \quad [u_R^{(1)}] = [u_R^{(2)}] = 0, \quad (2.8)$$

and the other is of the center of dilatation type,

$$[\sigma^{(1)}] = [\sigma_{RR}^{(2)}] = 0, \quad [u_R^{(1)}] = \tilde{B}_3 f_3(t)/R_0^2, \quad [u_R^{(2)}] = \tilde{B}_4 f_4(t)/R_0^2. \quad (2.9)$$

The field of the source (2.8) is described by the formulas

$$\begin{aligned} F_1(t) &= \frac{(\rho_{11}A_2 + \rho_{12})\tilde{B}_1 f_1(t) - (\rho_{12}A_2 + \rho_{22})\tilde{B}_2 f_2(t)}{3V_1^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)}, \\ F_2(t) &= -\frac{(\rho_{11}A_1 + \rho_{12})\tilde{B}_1 f_1(t) - (\rho_{12}A_1 + \rho_{22})\tilde{B}_2 f_2(t)}{3V_2^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)}, \end{aligned} \quad (2.10)$$

whereas the field of the source (2.9) is determined by the relations

$$\begin{aligned} F_1(t) &= -\frac{A_2\tilde{B}_3 f_3(t) - \tilde{B}_4 f_4(t)}{A_2 - A_1} - \frac{4L(\rho_{12}A_2 + \rho_{22})\tilde{B}_4 f_4(t)}{3V_1^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)}, \\ F_2(t) &= \frac{A_1\tilde{B}_3 f_3(t) - \tilde{B}_4 f_4(t)}{A_2 - A_1} + \frac{4L(\rho_{12}A_1 + \rho_{22})\tilde{B}_4 f_4(t)}{3V_2^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)}. \end{aligned} \quad (2.11)$$

Using relations (2.10) and (2.11), we can obtain expressions for the potentials in the fluid and elastic media. To this end, we use the formulas

$$\rho_{11} = \rho_s(1 - \varepsilon) + \varepsilon\rho_f(\alpha - 1), \quad \rho_{22} = \rho_f\varepsilon\alpha, \quad \rho_{12} = -\rho_f(\alpha - 1)\varepsilon, \quad (2.12)$$

which express the densities in the Biot model in terms of the porosity ε , the densities ρ_s and ρ_f of the elastic and fluid phases, and of the tortuosity α ($\alpha \geq 1$) of the porous space. In order to pass to sources of the second kind in the fluid medium, it is necessary to satisfy the conditions

$$\tilde{B}_2 = \tilde{B}_4 = 0, \quad \varepsilon = 1, \quad P = F = M = L = 0, \quad \tilde{R} = \lambda_1,$$

where λ_1 is Lamé's coefficient of the fluid. Under these conditions, we arrive at the equalities $V_1^2 = \tilde{R}/\rho_f$, $V_2 = 0$, $A_1 = 1$, $A_2 = \infty$, and the potentials for the sources of the center of compression and center of dilatation types in the fluid are expressed by the equalities

$$\begin{aligned} \phi_1(t) &= \tilde{B}_1 f_1(t)/(3\tilde{R}), \\ \phi_1(t) &= -\tilde{B}_3 f_3(t), \end{aligned} \quad (2.13)$$

respectively.

To pass to the elastic medium, it is necessary to satisfy the conditions

$$\tilde{B}_1 = \tilde{B}_3 = 0, \quad \varepsilon = 0, \quad M = \tilde{R} = 0, \quad F = \lambda_2, \quad L = \mu_2,$$

where λ_2 and μ_2 are Lamé's coefficients of the elastic phase. Under these conditions, $V_1^2 = P/\rho_s$, $V_2 = 0$, $A_1 = \alpha/(\alpha - 1) > 1$, $A_2 = 0$, $P = \lambda_2 + 2\mu_2$, and for the potentials in the elastic medium we obtain the representations

$$\begin{aligned} \phi_2(t) &= \tilde{B}_2 f_2(t)/(3P), \\ \phi_2(t) &= -\tilde{B}_4 f_4(t)(F + 2L/3)/P, \end{aligned} \quad (2.14)$$

which correspond to the sources of the center of compression and center of dilatation types, respectively. The first equalities in (2.13) and (2.14) coincide with the expressions in [1], where the fields excited by sources of the center of compression and center of dilatation types in elastic and fluid media were determined.

In formulas (2.10), the densities ρ_{11} , ρ_{22} , and ρ_{12} can be replaced by the coefficients of Hooke's law, using the equalities

$$\begin{aligned}\frac{\rho_{22} + \rho_{12}A_{2,1}}{V_{1,2}^2(\rho_{11}\rho_{22} - \rho_{12}^2)} &= \frac{MA_{2,1} + \tilde{R}}{P\tilde{R} - M^2}, \\ \frac{\rho_{11}A_{2,1} + \rho_{12}}{V_{1,2}^2(\rho_{11}\rho_{22} - \rho_{12}^2)} &= \frac{PA_{2,1} + M}{P\tilde{R} - M^2}.\end{aligned}\quad (2.15)$$

In view of relations (2.15), the functions $F_1(t)$ and $F_2(t)$, determining the wave field excited by the source of the center of compression type of the second kind, can be represented as follows:

$$\begin{aligned}F_1(t) &= \frac{(PA_2 + M)\tilde{B}_1 f_1(t) - (MA_2 + \tilde{R})\tilde{B}_2 f_2(t)}{3(A_2 - A_1)(P\tilde{R} - M^2)}, \\ F_2(t) &= -\frac{(PA_1 + M)\tilde{B}_1 f_1(t) - (MA_1 + \tilde{R})\tilde{B}_2 f_2(t)}{3(A_2 - A_1)(P\tilde{R} - M^2)}.\end{aligned}\quad (2.16)$$

If in equalities (2.16) we take $f_1(t) = f_2(t)$, $\tilde{B}_1 = -3\varepsilon/(4\pi)$, and $\tilde{B}_2 = -3(1 - \varepsilon)/(4\pi)$, then we obtain expressions that describe the wave field from a concentrated source of the center of compression type, which is specified by the right-hand sides of the equations [6].

The wave fields from both sources of the second kind can also be expressed in the cylindrical coordinates. Using formulas (1.6), (2.2), and (2.10), we represent the field of the source of the center of compression type by expressions (1.21), where

$$\begin{aligned}Z_1 &= \frac{K}{3(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)} \left\{ \frac{e^{-k|z|\alpha_1}}{V_1^2\alpha_1} \left[(\rho_{11}A_2 + \rho_{12})\tilde{B}_1 G_1 - (\rho_{12}A_2 + \rho_{22})\tilde{B}_2 G_2 \right] - \right. \\ &\quad \left. - \frac{e^{-k|z|\alpha_2}}{V_2^2\alpha_2} \left[(\rho_{11}A_1 + \rho_{12})\tilde{B}_1 G_1 - (\rho_{12}A_1 + \rho_{22})\tilde{B}_2 G_2 \right] \right\}, \\ Z_2 &= \frac{K}{3(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)} \left\{ \frac{e^{-k|z|\alpha_1}}{V_1^2\alpha_1} A_1 \left[(\rho_{11}A_2 + \rho_{12})\tilde{B}_1 G_1 - (\rho_{12}A_2 + \rho_{22})\tilde{B}_2 G_2 \right] - \right. \\ &\quad \left. - \frac{e^{-k|z|\alpha_2}}{V_2^2\alpha_2} A_2 \left[(\rho_{11}A_1 + \rho_{12})\tilde{B}_1 G_1 - (\rho_{12}A_1 + \rho_{22})\tilde{B}_2 G_2 \right] \right\}.\end{aligned}\quad (2.17)$$

The field of the source of the center of dilatation type can also be represented in a similar form, but, for brevity, these expressions are not presented.

§3. A CHECK OF THE SAINT-VENANT PRINCIPLE FOR THE BIOT MODEL

In this section, we show that the wave field from a point source of the center of compression type in a Biot medium can be represented as a superposition of wave fields excited by a pair of point sources of the type of vertical forces and by a source of the type of radial tangential forces. In the case of an elastic transversally-isotropic medium, this problem was solved in [1]. Its solution is consistent with the Saint-Venant principle. Here, we intend to extend these results to the Biot medium and to ascertain that the Saint-Venant principle is valid for such a medium.

In a homogeneous isotropic Biot medium given in the cylindrical coordinate system r, z, θ , consider the action of two auxiliary sources applied to the plane $z = 0$. The first source of the type of distributed normal pressure is given by the relations

$$[\sigma_{zz}^{(2)}] = \frac{\tilde{B}_2}{2\pi} f_2(t) \tilde{F}_n(r), \quad [\sigma^{(1)}] = \frac{\tilde{B}_1}{2\pi} f_1(t) \tilde{F}_n(r), \quad [\sigma_{rz}^{(2)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad (3.1)$$

which must hold for $z = 0$, where the symbol $[\cdot]$ expresses the jump of a function in square brackets on the plane $z = 0$, whereas the function $\tilde{F}_n(r)$ is given by

$$\tilde{F}_n(r) = \frac{n^2}{(1 + n^2 r^2)^{3/2}} \quad (3.2)$$

and satisfies the condition

$$\int_0^\infty \tilde{F}_n(r) r \, dr = 1. \quad (3.3)$$

In accordance with equalities (3.1) and (3.3), the resulting forces acting on the first and second phases in the plane $z = 0$ are equal to $\tilde{B}_1 f_1(t)$ and $\tilde{B}_2 f_2(t)$, respectively. Further, the parameter n specifies the distributions of these forces, and, for $n = \infty$, these forces prove to be applied to the origin of coordinates. A source of the type of nonuniform pressure on a fluid is unnatural, but we include it in our considerations for generality of formulation. The second source, involving tangential pressure, is determined by the following boundary conditions for $z = 0$:

$$\begin{aligned} [\sigma_{rz}^{(2)}] &= \frac{\tilde{B}}{2\pi} f(t) \phi_n(r), \quad [\sigma_{zz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_z^{(1)}] &= [u_z^{(2)}] = [u_r^{(2)}] = 0, \end{aligned} \quad (3.4)$$

where

$$\phi_n(r) = \frac{3n^4 r}{2(1 + n^2 r^2)^{5/2}} = -\frac{1}{2} \tilde{F}'_n(r). \quad (3.5)$$

In equalities (3.1) and (3.4), $\tilde{B}_1$, $\tilde{B}_2$, and $\tilde{B}$ are given coefficients, and the known functions $f_1(t)$, $f_2(t)$, and $f(t)$, which describe the time dependencies of the sources, are defined only for $t \geq 0$.

In terms of the potentials, the displacements and stresses occurring in conditions (3.1) and (3.4) are expressed as follows:

$$\begin{aligned} u_z^{(2)} &= \frac{\partial \varphi_2}{\partial z} - \frac{\partial \psi}{\partial r} - \frac{\psi}{r}, \\ u_z^{(1)} &= \frac{\partial \varphi_1}{\partial z} + \frac{\rho_{12}}{\rho_{22}} \left(\frac{\partial \psi}{\partial r} + \frac{\psi}{r} \right), \\ u_r^{(2)} &= \frac{\partial \varphi_2}{\partial r} + \frac{\partial \psi}{\partial z}, \\ \sigma^{(1)} &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ \sigma_{zz}^{(2)} &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_1}{\partial t^2} + 2L \left(\frac{\partial^2 \varphi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_2}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z} + \frac{1}{r} \frac{\partial \psi}{\partial z} \right), \\ \sigma_{rz}^{(2)} &= L \left(2 \frac{\partial^2 \varphi_2}{\partial r \partial z} + \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\psi}{r^2} \right). \end{aligned} \quad (3.6)$$

The potentials φ_1 , φ_2 , and ψ satisfy the equations

$$\begin{aligned} P \Delta \varphi_2 + M \Delta \varphi_1 &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ M \Delta \varphi_2 + \tilde{R} \Delta \varphi_1 &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} - \frac{\psi}{r^2} + \frac{\partial^2 \psi}{\partial z^2} &= \frac{\rho_{11} \rho_{22} - \rho_{12}^2}{\rho_{22} L} \frac{\partial^2 \psi}{\partial t^2}. \end{aligned} \quad (3.7)$$

In determining these potentials, it is convenient to consider the complex plane as a union of two half-planes, and to supplement the potentials with the superscript “-” in the half-plane $z > 0$ and with the superscript “+” in the half-plane $z < 0$. For both auxiliary sources, we seek the above potentials in the form of the following Fourier-Bessel and Mellin integrals:

$$\begin{aligned}
 \varphi_1^+ &= \int_0^\infty \frac{J_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (X_1^+ e^{kz\alpha_1} + X_2^+ e^{kz\alpha_2}) e^{kt\eta} d\eta, \\
 \varphi_2^+ &= \int_0^\infty \frac{J_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 X_1^+ e^{kz\alpha_1} + A_2 X_2^+ e^{kz\alpha_2}) e^{kt\eta} d\eta, \\
 \psi^+ &= - \int_0^\infty \frac{J_1(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y^+ e^{kt\eta+kz\beta} d\eta, \\
 \varphi_1^- &= \int_0^\infty \frac{J_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (X_1^- e^{-kz\alpha_1} + X_2^- e^{-kz\alpha_2}) e^{kt\eta} d\eta, \\
 \varphi_2^- &= \int_0^\infty \frac{J_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 X_1^- e^{-kz\alpha_1} + A_2 X_2^- e^{-kz\alpha_2}) e^{kt\eta} d\eta, \\
 \psi^- &= - \int_0^\infty \frac{J_1(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y^- e^{kt\eta-kz\beta} d\eta.
 \end{aligned} \tag{3.8}$$

The coefficients A_1 , A_2 and the radicals α_1 , α_2 are defined by (1.11) and (1.23) as previously, and the radical β is specified by the relations

$$\beta = \sqrt{1 + \frac{\eta^2}{V_3^2}}, \quad V_3 = \sqrt{\frac{\rho_{22}L}{\rho_{11}\rho_{22} - \rho_{12}^2}}. \tag{3.9}$$

In order to determine the displacement and stress fields, we must express the variables X_1^+ , X_2^+ , Y^+ , X_1^- , X_2^- , and Y^- in terms of the functions specifying the sources. To this end, we also represent the right-hand sides of conditions (3.1) and (3.4) by Fourier-Bessel and Mellin integrals, using the relations

$$\begin{aligned}
 \tilde{F}_n(r) &= \int_0^\infty J_0(kr) e^{-\frac{k}{n}} k dk, \\
 \tilde{\phi}_n(r) &= \frac{1}{2} \int_0^\infty J_1(kr) e^{-\frac{k}{n}} k^2 dk, \\
 G(k\eta) &= \int_0^\infty f(t) e^{-k\eta t} dt, \quad G_i(k\eta) = \int_0^\infty f_i(t) e^{-k\eta t} dt \quad (i = 1, 2).
 \end{aligned} \tag{3.10}$$

On solving the system of equations written on the basis of boundary conditions (3.1) and (3.4), we arrive at the expressions

$$X_1^- = -X_1^+ = \frac{(\rho_{11}A_2 + \rho_{12})\bar{B}_1 G_1 - (\rho_{12}A_2 + \rho_{22})\bar{B}_2 G_2}{4\pi\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$X_2^- = -X_2^+ = -\frac{(\rho_{11}A_1 + \rho_{12})\bar{B}_1G_1 - (\rho_{12}A_1 + \rho_{22})\bar{B}_2G_2}{4\pi\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$Y^- = Y^+ = \frac{\rho_{22}\bar{B}_2G_2 - \rho_{12}\bar{B}_1G_1}{4\pi\eta^2\beta(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

(3.11)

$$X_1^- = X_1^+ = \frac{(\rho_{12}A_2 + \rho_{22})\bar{B}kG}{4\pi\alpha_1\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$X_2^- = X_2^+ = -\frac{(\rho_{12}A_1 + \rho_{22})\bar{B}kG}{4\pi\alpha_2\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$Y^- = -Y^+ = -\frac{k\bar{B}\rho_{22}G}{4\pi\eta^2(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

(3.12)

which relate to the sources (3.1) and (3.4), respectively, and define the fields of potentials in accordance with (3.8). In identities (3.11) and (3.12), passage to the limit as $n \rightarrow \infty$ was carried out, after which we have $e^{-k/n} = 1$.

Now we consider the third auxiliary source, which is a pair of equal but oppositely directed forces applied to the points $r = 0, z = -R_0$ and $r = 0, z = R_0$. This source is given by the boundary conditions

$$[\sigma_{zz}^{(2)}] = \pm \frac{\bar{B}_2}{4\pi R_0} f_2(t) \tilde{F}_n(r),$$

$$[\sigma^{(1)}] = \pm \frac{\bar{B}_1}{4\pi R_0} f_1(t) \tilde{F}_n(r),$$

$$[\sigma_{rz}^{(2)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0,$$

(3.13)

where $z = \pm R_0$ and $n \rightarrow \infty$. In consequence of the similarity of the first and third auxiliary sources, we derive expressions for the potentials of the third source by using the expressions for the potentials of the first auxiliary source. If we denote the potentials in the case of the first auxiliary source by $\varphi_1(z)$, $\varphi_2(z)$, and $\psi(z)$, then the potentials for the third source are given by the expressions

$$\frac{\varphi_i(z - R_0) - \varphi_i(z + R_0)}{2R_0} \quad (i = 1, 2), \quad \frac{\psi(z - R_0) - \psi(z + R_0)}{2R_0}.$$

(3.14)

Upon passing to the limit as $R_0 \rightarrow 0$, expressions (3.14) reduce to

$$-\varphi'_i(z) \quad (i = 1, 2), \quad -\psi'(z),$$

(3.15)

respectively, and the fields of potentials corresponding to the third auxiliary source are represented by formulas (3.8) as well, where

$$X_1^- = X_1^+ = k\alpha_1 \frac{(\rho_{11}A_2 + \rho_{12})\bar{B}_1G_1 - (\rho_{12}A_2 + \rho_{22})\bar{B}_2G_2}{4\pi\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$X_2^- = X_2^+ = -k\alpha_2 \frac{(\rho_{11}A_1 + \rho_{12})\bar{B}_1G_1 - (\rho_{12}A_1 + \rho_{22})\bar{B}_2G_2}{4\pi\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$Y^- = -Y^+ = k \frac{\rho_{22}\bar{B}_2G_2 - \rho_{12}\bar{B}_1G_1}{4\pi\eta^2(\rho_{11}\rho_{22} - \rho_{12}^2)}.$$

(3.16)

Now we consider the case where the medium is simultaneously excited by the second and third auxiliary sources. Summing the corresponding expressions in (3.12) and (3.16), we arrive at the formulas

$$X_1^- = X_1^+ = k \frac{(\rho_{12}A_2 + \rho_{22})(\bar{B}G - \bar{B}_2G_2\alpha_1^2) + (\rho_{11}A_2 + \rho_{12})\bar{B}_1G_1\alpha_1^2}{4\pi\alpha_1\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$X_2^- = X_2^+ = -k \frac{(\rho_{12}A_1 + \rho_{22})(\bar{B}G - \bar{B}_2G_2\alpha_2^2) + (\rho_{11}A_1 + \rho_{12})\bar{B}_1G_1\alpha_2^2}{4\pi\alpha_2\eta^2(A_2 - A_1)(\rho_{11}\rho_{22} - \rho_{12}^2)},$$

$$Y^- = -Y^+ = k \frac{\rho_{22}(\bar{B}_2G_2 - \bar{B}G) - \rho_{12}\bar{B}_1G_1}{4\pi\eta^2(\rho_{11}\rho_{22} - \rho_{12}^2)}.$$

(3.17)

We compare the wave field determined by (3.8) and (3.17) with the spherically symmetric field excited by the source of the center of compression type of the second kind. In order that the wave field be spherically symmetric, the condition $\psi = 0$ must be satisfied, which leads to the relations

$$\bar{B}_2 G_2 = \bar{B} G, \quad \bar{B}_1 G_1 = 0. \quad (3.18)$$

With account for identities (3.18), formulas (3.17) can be represented in the form

$$\begin{aligned} X_1^- &= X_1^+ = -k \frac{(\rho_{12} A_2 + \rho_{22}) \bar{B}_2 G_2}{4\pi \alpha_1 V_1^2 (A_2 - A_1)(\rho_{11} \rho_{22} - \rho_{12}^2)}, \\ X_2^- &= X_2^+ = k \frac{(\rho_{12} A_1 + \rho_{22}) \bar{B}_2 G_2}{4\pi \alpha_2 V_2^2 (A_2 - A_1)(\rho_{11} \rho_{22} - \rho_{12}^2)}. \end{aligned} \quad (3.19)$$

If in formulas (2.17) we take

$$\tilde{B}_1 = 0, \quad \tilde{B}_2 = 3\bar{B}_2/(4\pi), \quad (3.20)$$

then relations (2.17) and (3.19) describe the same wave fields. Thus, the source of the center of compression type of the second kind is equivalent to a pair of the corresponding vertical forces and a certain source of the center of tangent pressure type, provided that all of the above sources act directly only on the elastic phase. If, instead of the second formula in (2.8), we write the relation $[\sigma_{RR}^{(2)}] = 3\tilde{B}_2 f_2(t)/(4\pi R_0^3)$, which shows that the jump of pressure is inversely proportional to the volume of the ball to the boundary of which the action is applied, then relations (3.20) are replaced by the identities $\tilde{B}_1 = 0$, $\tilde{B}_2 = \bar{B}_2$.

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