

JUMPS OF DISPLACEMENTS AND STRESSES AS SEISMIC SOURCES IN BIOT'S MEDIUM

L. A. Molotkov and A. V. Bakulin

UDC 517.934; 550.34

Wave fields excited in a homogeneous isotropic Biot medium by point sources described in terms of discontinuities of displacements and stresses are determined. The results are represented in the form of relations involving Fourier-Bessel or Mellin integrals and in the form of Stokes-type formulas. The interrelations between these representations are established. Among all possible point sources exciting Biot's medium, the elementary sources, in terms of which any complicated linear source can be described, are selected. The special case where the wave fields in the two phases of the Biot medium are independent of one another is considered, and the corresponding sources in the Biot medium are compared with the known sources in elastic and fluid media. Bibliography: 14 titles.

In the context of the development of the theory of seismic wave propagation, investigations of wave processes in porous and fractured media based on effective models and, in particular, on Biot's model are of considerable interest. In order to extract detailed information on wave propagation from Biot's model, it is necessary to consider different sources. Some sources for this model were already defined in [1-4]. This paper is devoted to defining new sources, to establishing interrelations among sources, and to selecting elementary sources, based on which wave fields of more complicated sources in an unbounded Biot medium can be described. This paper is specific in that sources are described in terms of boundary conditions involving jumps of stresses and displacements. The resulting wave fields are represented in two forms, namely, by relations similar to the Stokes formulas and by equalities using the Fourier-Bessel and Laplace transforms. The first representation is convenient, in particular, in investigating wave fields in the near and far zones, whereas the second representation is needed in studying reflection and refraction processes on interfaces. In this paper, in order to establish the relations between the two representations, a number of integral identities is derived. Since Biot's medium is a generalization of both elastic and fluid media, for which the sources are already known [5-7], we ascertain that the sources found in the Biot medium correspond to the sources in elastic and fluid media.

1. BASIC EQUATIONS OF A HOMOGENEOUS ISOTROPIC BIOT MEDIUM

Let an unbounded, homogeneous, isotropic, two-phase, Biot medium [8] be given, in which the superscripts 1 and 2 refer to the fluid and elastic phases, respectively. The displacement vectors $\vec{u}^{(1)}$ and $\vec{u}^{(2)}$ are expressed by the equalities

$$\begin{aligned}\vec{u}^{(1)} &= \text{grad } \varphi_1 - \frac{\rho_{12}}{\rho_{22}} \text{rot } \vec{\psi}, \\ \vec{u}^{(2)} &= \text{grad } \varphi_2 + \text{rot } \vec{\psi},\end{aligned}\tag{1.1}$$

where the potentials φ_1 , φ_2 , and $\vec{\psi}$ satisfy the equations

$$\begin{aligned}P\Delta\varphi_2 + M\Delta\varphi_1 &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ M\Delta\varphi_2 + \bar{R}\Delta\varphi_1 &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ \Delta\vec{\psi} &= \frac{\delta}{\rho_{22}L} \frac{\partial^2 \vec{\psi}}{\partial t^2},\end{aligned}\tag{1.2}$$

Translated from *Zapiski Nauchnykh Seminarov POMI*, Vol. 239, 1997, pp. 164-196. Original article submitted July 15, 1996.

and $\delta = \rho_{11}\rho_{22} - \rho_{12}^2$. In relations (1.1) and (1.2), the parameters P , M , $\tilde{R}$, and L are moduli of the medium, and the quantities ρ_{11} , ρ_{22} , and ρ_{12} are defined by the equalities

$$\rho_{11} = \rho_s(1 - \varepsilon) + \varepsilon\rho_f(\alpha - 1), \quad \rho_{12} = -\rho_f\varepsilon(\alpha - 1), \quad \rho_{22} = \rho_f\varepsilon\alpha, \quad (1.3)$$

in which ρ_s and ρ_f are the densities of the elastic and fluid phases; ε is porosity, and α is the tortuosity of pores ($\alpha \geq 1$). The tensors of stresses $\sigma^{(1)}$ and $\sigma_{ik}^{(2)}$ ($i, k = x, y, z$) are expressed in terms of the displacements via the formulas of Hooke's law

$$\begin{aligned} \sigma_{xx}^{(2)} &= P \frac{\partial u_x^{(2)}}{\partial x} + A \frac{\partial u_y^{(2)}}{\partial y} + A \frac{\partial u_z^{(2)}}{\partial z} + M\theta, \\ \sigma_{yy}^{(2)} &= A \frac{\partial u_x^{(2)}}{\partial x} + P \frac{\partial u_y^{(2)}}{\partial y} + A \frac{\partial u_z^{(2)}}{\partial z} + M\theta, \\ \sigma_{zz}^{(2)} &= A \frac{\partial u_x^{(2)}}{\partial x} + A \frac{\partial u_y^{(2)}}{\partial y} + P \frac{\partial u_z^{(2)}}{\partial z} + M\theta, \\ \sigma^{(1)} &= M \frac{\partial u_x^{(2)}}{\partial x} + M \frac{\partial u_y^{(2)}}{\partial y} + M \frac{\partial u_z^{(2)}}{\partial z} + \tilde{R}\theta, \\ \sigma_{yz}^{(2)} &= L \left(\frac{\partial u_y^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial y} \right), \quad \sigma_{xz}^{(2)} = L \left(\frac{\partial u_x^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial x} \right), \\ \sigma_{xy}^{(2)} &= L \left(\frac{\partial u_x^{(2)}}{\partial y} + \frac{\partial u_y^{(2)}}{\partial x} \right), \quad \theta = \frac{\partial u_x^{(1)}}{\partial x} + \frac{\partial u_y^{(1)}}{\partial y} + \frac{\partial u_z^{(1)}}{\partial z}, \end{aligned} \quad (1.4)$$

where $A = P - 2L$. In the medium considered, waves in the elastic and fluid phases are interrelated. These waves propagate independently and are described by the equations of elasticity theory and of hydroacoustics, respectively, only in the case where

$$\rho_{12} = 0, \quad M = 0. \quad (1.5)$$

2. THE DESCRIPTION OF ELEMENTARY SOURCES AND OF THE RELATED WAVE FIELDS

The purpose of this paper is to establish sources that can excite the medium in question. Among these sources we select the elementary sources, in terms of which other sources and the wave fields excited by them can easily be described. The first of the elementary sources is defined in the cylindrical system of coordinates r, θ, z by the relations

$$[\sigma_{zz}^{(2)}] = \frac{\delta(r)f(t)}{2\pi r}, \quad [\sigma^{(1)}] = [\sigma_{rz}^{(2)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad (2.1)$$

where $\delta(r)$ is the Dirac function, and the symbol $[\]$ denotes the jump of the function in square brackets on the plane $z = 0$. The right-hand side of the first equation in (2.1) expresses a concentrated external force acting along the z axis and applied to the solid phase. For this source, the zero initial conditions for $t = 0$

$$\vec{u}^{(1)} = \vec{u}^{(2)} = 0, \quad \partial \vec{u}^{(1)} / \partial t = \partial \vec{u}^{(2)} / \partial t = 0 \quad (2.2)$$

are satisfied, and the function $f(t)$, specifying the time dependence of the source, vanishes for $t < 0$.

In order to find the solutions of Eqs. (1.2) that satisfy boundary conditions (2.1) and initial conditions (2.2), we pass in Eqs. (1.2) and (1.4) to the cylindrical coordinates. In this coordinate system, it is convenient to represent the vector potential $\vec{\psi}$ in the form

$$\vec{\psi} = \psi_\theta \vec{\theta}_1. \quad (2.3)$$

With regard for (2.3), the displacements and stresses are expressed as follows:

$$\begin{aligned}
u_z^{(1)} &= \frac{\partial \varphi_1}{\partial z} - \frac{\rho_{12}}{\rho_{22}} \left(\frac{\partial \psi_\theta}{\partial r} + \frac{\psi_\theta}{r} \right), & u_r^{(1)} &= \frac{\partial \varphi_1}{\partial r} + \frac{\rho_{12}}{\rho_{22}} \frac{\partial \psi_\theta}{\partial z}, \\
u_z^{(2)} &= \frac{\partial \varphi_2}{\partial z} + \frac{\partial \psi_\theta}{\partial r} + \frac{\psi_\theta}{r}, & u_r^{(2)} &= \frac{\partial \varphi_2}{\partial r} - \frac{\partial \psi_\theta}{\partial z}, \\
\sigma^{(1)} &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}, \\
\sigma_{zz}^{(2)} &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_1}{\partial t^2} + 2L \left(\frac{\partial^2 \varphi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_2}{\partial r} - \frac{\partial^2 \psi_\theta}{\partial r \partial z} - \frac{1}{r} \frac{\partial \psi_\theta}{\partial z} \right), \\
\sigma_{rz}^{(2)} &= L \left(2 \frac{\partial^2 \varphi_2}{\partial r \partial z} - \frac{\partial^2 \psi_\theta}{\partial z^2} + \frac{\partial^2 \psi_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_\theta}{\partial r} - \frac{\psi_\theta}{r^2} \right).
\end{aligned} \tag{2.4}$$

By applying the Fourier-Bessel and Laplace integral transforms and using the results of [4], for the potentials corresponding to the source (2.1) we obtain the representations

$$\begin{aligned}
\varphi_1 &= \text{sign } z \int_0^\infty \frac{J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left(X_1 e^{-k|z|\alpha_1} + X_2 e^{-k|z|\alpha_2} \right) e^{k\eta} d\eta, \\
\varphi_2 &= \text{sign } z \int_0^\infty \frac{J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left(A_1 X_1 e^{-k|z|\alpha_1} + A_2 X_2 e^{-k|z|\alpha_2} \right) e^{k\eta} d\eta, \\
\psi_\theta &= \int_0^\infty \frac{J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y e^{-k|z|\beta} e^{k\eta} d\eta,
\end{aligned} \tag{2.5}$$

where

$$\begin{aligned}
X_1 &= -\frac{(\rho_{12}A_2 + \rho_{22})F(k\eta)}{4\pi\eta^2(A_2 - A_1)\delta}, & X_2 &= \frac{(\rho_{12}A_1 + \rho_{22})F(k\eta)}{4\pi\eta^2(A_2 - A_1)\delta}, & Y &= \frac{\rho_{22}F(k\eta)}{4\pi\eta^2\beta\delta}, \\
F(k\eta) &= \int_0^\infty f(t) e^{-k\eta t} dt.
\end{aligned} \tag{2.6}$$

The quantities A_1 , A_2 , δ , α_1 , α_2 , and β occurring in formulas (2.5) and (2.6) are specified by the equalities

$$\begin{aligned}
A_{1,2} &= \frac{\rho_{12}v_{1,2}^2 - M}{P - \rho_{11}v_{1,2}^2} = \frac{\rho_{22}v_{1,2}^2 - \bar{R}}{M - \rho_{12}v_{1,2}^2}, \\
\delta &= \rho_{11}\rho_{22} - \rho_{12}^2, & \alpha_1 &= \sqrt{1 + \frac{\eta^2}{v_1^2}}, & \alpha_2 &= \sqrt{1 + \frac{\eta^2}{v_2^2}}, & \beta &= \sqrt{1 + \frac{\eta^2}{v_3^2}},
\end{aligned} \tag{2.7}$$

where v_1 , v_2 , and v_3 are the velocities of propagation of two longitudinal waves and one transverse wave, determined by the relations

$$v_{1,2}^2 = \frac{P\rho_{22} + \bar{R}\rho_{11} - 2M\rho_{12} \pm \sqrt{(P\rho_{22} + \bar{R}\rho_{11} - 2M\rho_{12})^2 - 4\delta(P\bar{R} - M^2)}}{2\delta}, \quad v_3 = \sqrt{L\rho_{22}/\delta}. \tag{2.8}$$

In order to satisfy the zero initial conditions (2.2), we draw cuts from the branch points $\eta = \pm iv_1$, $\eta = \pm iv_2$, and $\eta = \pm iv_3$ into the left half-plane η and specify the principal branches by the conditions $\alpha_1(0) = \alpha_2(0) = \beta(0) = 1$.

In order to analyze and evaluate wave fields in Biot's medium, it is expedient to derive Stokes-type formulas for the potentials φ_1 , φ_2 , and $\bar{\psi}$. To this end, we use the relation [9]

$$\frac{e^{-\gamma R}}{R} = \int_0^\infty k J_0(kr) \frac{e^{-|z|\sqrt{k^2+\gamma^2}}}{\sqrt{k^2+\gamma^2}} dk, \quad (2.9)$$

where $R = \sqrt{r^2 + z^2}$ is the spherical radius. Formula (2.9) implies the identity [4]

$$\frac{f(t - R/v)}{R} = \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta) e^{k(t\eta-|z|\alpha)}}{\alpha} d\eta, \quad (2.10)$$

which holds for an arbitrary function $f(t)$ defined for $t \geq 0$. Here,

$$\alpha = \sqrt{1 + \frac{\eta^2}{v^2}}, \quad F(k\eta) = \int_0^\infty f(t) e^{-k\eta t} d\eta. \quad (2.11)$$

Integrating (2.10) with respect to t twice and differentiating the result obtained with respect to z and r , we arrive at the formulas

$$\begin{aligned} \int_0^\infty \frac{J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta) e^{k(t\eta-|z|\alpha)}}{\eta^2} d\eta &= -\text{sign } z \left(\frac{\partial}{\partial z} \frac{1}{R} \right) \int_{R/v}^t \tau f(t-\tau) d\tau, \\ \int_0^\infty \frac{J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta) e^{k(t\eta-|z|\alpha)}}{\alpha \eta^2} d\eta &= - \left(\frac{\partial}{\partial r} \frac{1}{R} \right) \int_{R/v}^t \tau f(t-\tau) d\tau. \end{aligned} \quad (2.12)$$

Using formulas (2.12), one can reduce the expressions for the potentials φ_1 , φ_2 , and ψ_θ in the Biot medium to the form

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \frac{1}{4\pi(A_2 - A_1)\delta} \left(\frac{\partial}{\partial z} \frac{1}{R} \right) \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} (\rho_{12}A_2 + \rho_{22}) \int_{R/v_1}^t \tau f(t-\tau) d\tau \right. \\ &\quad \left. - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} (\rho_{12}A_1 + \rho_{22}) \int_{R/v_2}^t \tau f(t-\tau) d\tau \right], \\ \psi_\theta &= -\frac{\rho_{22}}{4\pi\delta} \left(\frac{\partial}{\partial r} \frac{1}{R} \right) \int_{R/v_3}^t \tau f(t-\tau) d\tau. \end{aligned} \quad (2.13)$$

In order to pass to Cartesian coordinates, we replace ψ_θ by the potentials

$$\psi_x = -\psi_\theta \sin \theta, \quad \psi_y = \psi_\theta \cos \theta, \quad \psi_z = 0, \quad (2.14)$$

for which we obtain the expressions

$$\psi_x = \frac{\rho_{22}}{4\pi\delta} \left(\frac{\partial}{\partial y} \frac{1}{R} \right) \int_{R/v_3}^t \tau f(t-\tau) d\tau, \quad \psi_y = -\frac{\rho_{22}}{4\pi\delta} \left(\frac{\partial}{\partial x} \frac{1}{R} \right) \int_{R/v_3}^t \tau f(t-\tau) d\tau. \quad (2.15)$$

The equalities (2.13) and (2.15) are the Stokes-type formulas for the potentials in Biot's medium excited by the source (2.1).

Such a source in the fluid phase is not considered because it is inconsistent with the physical view of wave fields in fluids or fluid phase [4]. Indeed, a wave field in a fluid (or a fluid phase) is necessarily potential, whereas any source of the type of a concentrated force necessarily contains a rotational component.

The second elementary source acting in Biot's model is a center of pressure applied to the fluid phase. This source is given on a sphere $R = R_0$ by the relations

$$[\sigma^{(1)}] = \frac{3f(t)}{4\pi R_0^3}, \quad [\sigma_{RR}^{(2)}] = 0, \quad [u_R^{(1)}] = [u_R^{(2)}] = 0, \quad (2.16)$$

where the symbol $[\]$ denotes the jump on the sphere $R = R_0$ of the function in square brackets. For such a source, we determine the wave field and pass to the limit as $R_0 \rightarrow 0$. A detailed investigation of the source (2.16), including passage to the limit, was carried out in [4]. The resulting wave field is described by the relations [4]

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{4\pi(A_2 - A_1)\delta R} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{\rho_{11}A_2 + \rho_{12}}{v_1^2} f\left(t - \frac{R}{v_1}\right) - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\rho_{11}A_1 + \rho_{12}}{v_2^2} f\left(t - \frac{R}{v_2}\right) \right], \quad (2.17)$$

$$\psi_\theta = 0.$$

By applying the integral formula (2.10), for the potentials φ_1 and φ_2 we obtain the representations

$$\begin{aligned} \varphi_1 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (X_1 e^{-k|z|\alpha_1} + X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \varphi_2 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 X_1 e^{-k|z|\alpha_1} + A_2 X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \end{aligned} \quad (2.18)$$

where

$$X_1 = \frac{(\rho_{11}A_2 + \rho_{12})F(k\eta)}{4\pi v_1^2(A_2 - A_1)\delta\alpha_1}, \quad X_2 = -\frac{(\rho_{11}A_1 + \rho_{12})F(k\eta)}{4\pi v_2^2(A_2 - A_1)\delta\alpha_2}. \quad (2.19)$$

Under conditions (1.5), the source (2.1) excites waves only in the elastic phase, whereas waves excited by the source (2.16) propagate only in the fluid phase. Under the additional assumption

$$P\rho_{22} > \tilde{R}\rho_{11}, \quad (2.20)$$

which almost always holds, the equalities

$$v_1^2 = P/\rho_{11}, \quad v_2^2 = \tilde{R}/\rho_{22}, \quad A_1 = \infty, \quad A_2 = 0 \quad (2.21)$$

are valid. Taking into account the relations $A_1 = \infty$, $A_2 = 0$, and $\rho_{12} = 0$, we arrive at the expressions

$$\begin{aligned} \varphi_1 &= 0, \quad \varphi_2 = \frac{\text{sign } z}{4\pi\rho_{11}} \int_0^\infty \frac{J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta)}{\eta^2} e^{k(t\eta - |z|\alpha_1)} d\eta, \\ \psi_\theta &= \frac{1}{4\pi\rho_{11}} \int_0^\infty \frac{J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta)}{\beta\eta^2} e^{k(t\eta - |z|\beta)} d\eta, \end{aligned} \quad (2.22)$$

which are based on formulas (2.5)–(2.7) and describe elastic waves in the skeleton excited by the source (2.1). In Stokes form, the same quantities can be represented as follows:

$$\varphi_1 = 0, \quad \varphi_2 = \frac{z}{4\pi\rho_{11}R^3} \int_{R/v_1}^t \tau f(t-\tau) d\tau, \quad \psi_\theta = \frac{r}{4\pi\rho_{11}R^3} \int_{R/v_3}^t \tau f(t-\tau) d\tau. \quad (2.23)$$

Relations (2.22) and (2.23) describe the potentials of a wave field in an elastic medium with density ρ_{11} and velocities of propagation v_1 and v_3 . Relations (2.23) correspond to the Stokes-type formulas for an elastic medium excited by a concentrated force (see [6, formulas (4.21), (4.22)]).

Upon similar passage to the limit in formulas (2.17) and (2.18), we obtain the equalities

$$\varphi_1 = \frac{1}{4\pi\rho_{22}v_2^2 R} f\left(t - \frac{R}{v_2}\right), \quad \varphi_2 = 0, \quad \psi_\theta = 0; \quad (2.24)$$

$$\varphi_1 = \frac{1}{4\pi\rho_{22}v_2^2} \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F(k\eta) \frac{e^{k(t\eta-|z|\alpha_2)}}{\alpha_2} d\eta, \quad \varphi_2 = 0, \quad \psi_\theta = 0, \quad (2.25)$$

which describe the wave field in a fluid with density ρ_{22} and velocity of propagation v_2 that is excited by a source of the center of pressure type. Formulas (2.24) and (2.25) coincide with the corresponding relations in [10]. To conclude this section, we note that if, under conditions (1.5), the inequality $P_{\rho_{22}} < \bar{R}_{\rho_{11}}$, the reverse of (2.20), is satisfied, then the wave fields in Biot's medium also turn into wave fields in elastic and fluid media.

3. ON THE SOURCES REPRESENTED BY FORCE COUPLES ACTING ON THE ELASTIC PHASE

3.1. Assume that Biot's medium is excited by two oppositely directed forces applied to the points $r = 0$, $z = R_0$ and $r = 0$, $z = -R_0$. Both sources are specified by the equalities

$$[\sigma_{zz}^{(2)}] = \pm \frac{\delta(\tau)f(t)}{4\pi r R_0}, \quad [\sigma_{zr}^{(2)}] = [\sigma^{(1)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad z = \pm R_0, \quad (3.1)$$

where the symbol $[]$ means the jump of the function in square brackets on the corresponding plane. In the wave field satisfying conditions (3.1) and (2.2), it is necessary to pass to the limit as $R_0 \rightarrow 0$. This source was investigated in [4], where it was shown that in order to pass from the wave field of the source (2.1) to the wave field of the source (3.1), it is sufficient to differentiate expressions (2.5) and (2.13) with respect to z and to change signs.

In accordance with this rule, the potentials will be given by the Stokes-type formulas

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \frac{\rho_{12}A_2 + \rho_{22}}{4\pi(A_2 - A_1)\delta R^3} \left\{ \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{r^2 - 2z^2}{R^2} \int_{R/v_1}^t \tau f(t-\tau) d\tau - \frac{z^2}{v_1^2} f\left(t - \frac{R}{v_1}\right) \right] \right. \\ &\quad \left. - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{r^2 - 2z^2}{R^2} \int_{R/v_2}^t \tau f(t-\tau) d\tau - \frac{z^2}{v_2^2} f\left(t - \frac{R}{v_2}\right) \right] \right\}, \\ \psi_x &= -\frac{\rho_{22}yz}{4\pi\delta R^3} \left[\frac{3}{R^2} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{1}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right], \\ \psi_y &= \frac{\rho_{22}xz}{4\pi\delta R^3} \left[\frac{3}{R^2} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{1}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right]. \end{aligned} \quad (3.2)$$

In terms of Fourier-Bessel and Mellin integrals, the expressions for the potentials are of the form

$$\begin{aligned}\varphi_1 &= - \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (\alpha_1 X_1 e^{-k|z|\alpha_1} + \alpha_2 X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \varphi_2 &= - \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 \alpha_1 X_1 e^{-k|z|\alpha_1} + A_2 \alpha_2 X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \psi_\theta &= -\text{sign } z \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \beta Y e^{-k|z|\beta} e^{kt\eta} d\eta.\end{aligned}\quad (3.3)$$

Here, the functions X_1 , X_2 , and Y are determined by relations (2.6).

The approach based on superposing wave fields excited by sources of the same type can also be used in determining fields excited by dipoles and quadrupoles of general type. We confine ourselves to considering the cases of two and three couples of oppositely directed forces.

3.2. Assume that Biot's medium is excited by three force couples acting along the three coordinate axes

$$\begin{aligned}[\sigma_{zz}^{(2)}] &= \pm \frac{\delta(x)\delta(y)f(t)}{2R_0}, \quad [\sigma_{zz}^{(2)}] = [\sigma_{yz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_z^{(1)}] &= [u_z^{(2)}] = [u_x^{(2)}] = [u_y^{(2)}] = 0, \quad z = \pm R_0; \\ [\sigma_{xx}^{(2)}] &= \pm \frac{\delta(y)\delta(z)f(t)}{2R_0}, \quad [\sigma_{xy}^{(2)}] = [\sigma_{xz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_x^{(1)}] &= [u_x^{(2)}] = [u_y^{(2)}] = [u_z^{(2)}] = 0, \quad x = \pm R_0; \\ [\sigma_{yy}^{(2)}] &= \pm \frac{\delta(x)\delta(z)f(t)}{2R_0}, \quad [\sigma_{yz}^{(2)}] = [\sigma_{xy}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_y^{(1)}] &= [u_y^{(2)}] = [u_x^{(2)}] = [u_z^{(2)}] = 0, \quad y = \pm R_0.\end{aligned}\quad (3.4)$$

The first force couple defined by conditions (3.4) coincides with the forces given by relations (3.1) and gives rise to the wave field that, upon passage to the limit as $R_0 \rightarrow 0$, is described by formulas (3.2). In order to express the potentials of the wave fields excited by the second and third force couples, we use cyclic permutation. Without presenting the explicit expressions for the latter potentials, we will find the composition of the potentials for all three force couples. Under this composition, the components of the potential $\tilde{\psi}$ and the integral terms in formulas (3.2) will be cancelled. As a result we will obtain the equalities

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = - \frac{1}{4\pi(A_2 - A_1)\delta R} \left[\left(\frac{1}{A_1} \right) \frac{\rho_{12}A_2 + \rho_{22}}{v_1^2} f\left(t - \frac{R}{v_1}\right) - \left(\frac{1}{A_2} \right) \frac{\rho_{12}A_1 + \rho_{22}}{v_2^2} f\left(t - \frac{R}{v_2}\right) \right], \quad (3.5)$$

which determine the wave field excited by the source (3.4).

Formulas (3.5) coincide with the known relations [4] for the potentials in the case of the source

$$[\sigma_{RR}^{(2)}] = \frac{3f(t)}{4\pi R_0^3}, \quad [\sigma^{(1)}] = 0, \quad [u_R^{(1)}] = [u_R^{(2)}] = 0 \quad (3.6)$$

of the center-of-pressure type acting on the elastic phase. In [4], the jumps were given for $R = R_0$, and in the wave field, passage to the limit as $R_0 \rightarrow 0$ was performed. Thus, the concentrated sources that are represented in the form of three momentless force couples acting on the elastic phase and those represented in the form of a center of pressure prove to be equivalent. In terms of integral transforms, the potentials (3.5) are given by relations (2.18), where

$$X_1 = - \frac{(\rho_{12}A_2 + \rho_{22})F(k\eta)}{4\pi(A_2 - A_1)\delta v_1^2 \alpha_1}, \quad X_2 = \frac{(\rho_{12}A_1 + \rho_{22})F(k\eta)}{4\pi(A_2 - A_1)\delta v_2^2 \alpha_2}. \quad (3.7)$$

3.3. Now we consider the source that consists of two force couples acting along the x and y axes and is described by relations (3.4) for $x = \pm R_0$ and $y = \pm R_0$. By using cyclic permutations in formulas (3.2), adding the expressions for the potentials together, and passing to the limit as $R_0 \rightarrow 0$, we obtain the relations

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \frac{1}{4\pi(A_2 - A_1)\delta} \left\{ (\rho_{12}A_2 + \rho_{22}) \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{2z^2 - \tau^2}{R^5} \int_{R/v_1}^t \tau f(t - \tau) d\tau - \frac{\tau^2}{v_1^2 R^3} f\left(t - \frac{R}{v_1}\right) \right] \right. \\ &\quad \left. - (\rho_{12}A_1 + \rho_{22}) \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{2z^2 - \tau^2}{R^5} \int_{R/v_2}^t \tau f(t - \tau) d\tau - \frac{\tau^2}{v_2^2 R^3} f\left(t - \frac{R}{v_2}\right) \right] \right\}, \\ \psi_\theta &= -\frac{\rho_{22}rz}{4\pi\delta R^3} \left[\frac{3}{R^2} \int_{R/v_3}^t \tau f(t - \tau) d\tau + \frac{1}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right], \end{aligned} \quad (3.8)$$

which determine the wave field. Using Fourier-Bessel and Mellin integrals, we can rewrite relations (3.8) in the form

$$\begin{aligned} \varphi_1 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (X_1 e^{-k|z|\alpha_1} + X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \varphi_2 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 X_1 e^{-k|z|\alpha_1} + A_2 X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \psi_\theta &= \text{sign } z \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y e^{-k|z|\beta} e^{kt\eta} d\eta, \end{aligned} \quad (3.9)$$

where

$$X_1 = \frac{\rho_{12}A_2 + \rho_{22}}{4\pi(A_2 - A_1)\delta\alpha_1\eta^2}, \quad X_2 = -\frac{\rho_{12}A_1 + \rho_{22}}{4\pi(A_2 - A_1)\delta\alpha_2\eta^2}, \quad Y = -\frac{\rho_{22}}{4\pi\delta\eta^2}. \quad (3.10)$$

According to [4], relations (3.9) and (3.10) describe the wave field excited by a source of the type of a center of radial tangential forces on the plane $z = 0$. This source is given on the plane $z = 0$ by the relations

$$[\sigma_{rz}^{(2)}] = \frac{3n^4 r f(t)}{4\pi(1 + n^2 r^2)^{5/2}}, \quad [\sigma_{zz}^{(2)}] = [\sigma_{\theta z}^{(2)}] = [\sigma^{(1)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad (3.11)$$

and, in the wave field constructed, passage to the limit as $n \rightarrow \infty$ must be carried out. Thus, the action of the two force couples considered is equivalent to that of a center of radial tangential forces on a plane.

3.4. Now we turn to the consideration of a force couple with a moment represented schematically in Fig. 1 and defined by the relations

$$[\sigma_{zz}^{(2)}] = \frac{\delta(x - R_0)\delta(y)f(t)}{4\pi R_0} - \frac{\delta(x + R_0)\delta(y)f(t)}{4\pi R_0}, \quad (3.12)$$

$$[\sigma_{zz}^{(2)}] = [\sigma_{zy}^{(2)}] = [\sigma^{(1)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_x^{(2)}] = [u_y^{(2)}] = 0$$

on the plane $z = 0$. In the wave field satisfying conditions (3.12) it is necessary to carry out passage to the limit as $R_0 \rightarrow 0$. Using the same line of reasoning as in deriving the wave field excited by a force couple, we conclude that, in this case, in order to determine the potentials, it is sufficient to differentiate the left-hand

sides of (2.13) and (2.15) with respect to x and to change signs. As a result, for the potentials of the wave field we obtain the expressions

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = - \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{\rho_{12}A_2 + \rho_{22}}{4\pi(A_2 - A_1)\delta} \left[\frac{3xz}{R^5} \int_{R/v_1}^t \tau f(t-\tau) d\tau + \frac{xz}{v_1^2 R^3} f\left(t - \frac{R}{v_1}\right) \right] \\ + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\rho_{12}A_1 + \rho_{22}}{4\pi(A_2 - A_1)\delta} \left[\frac{3xz}{R^5} \int_{R/v_2}^t \tau f(t-\tau) d\tau + \frac{xz}{v_2^2 R^3} f\left(t - \frac{R}{v_2}\right) \right]; \quad (3.13)$$

$$\psi_x = -\frac{\rho_{22}}{4\pi\delta} \left[\frac{3xy}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{xy}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right], \quad (3.14)$$

$$\psi_y = \frac{\rho_{22}}{4\pi\delta} \left[\frac{2x^2 - y^2 - z^2}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{x^2}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right].$$

3.5. Consider another force couple represented in Fig. 2 and given by the conditions

$$[\sigma_{xx}^{(2)}] = \frac{\delta(z - R_0)\delta(y)f(t)}{4\pi R_0} - \frac{\delta(z + R_0)\delta(y)f(t)}{4\pi R_0}, \quad (3.15)$$

$$[\sigma_{zz}^{(2)}] = [\sigma_{xy}^{(2)}] = [\sigma^{(1)}] = 0, \quad [u_x^{(1)}] = [u_x^{(2)}] = [u_y^{(2)}] = [u_z^{(2)}] = 0$$

on the plane $x = 0$. In order to determine the wave field as $R_0 \rightarrow 0$, in formulas (2.13) and (2.15) it is necessary to perform the cyclic permutation that replaces the coordinates x, y, z by the coordinates y, z, x and to take the expressions obtained for the derivatives with respect to z with the opposite signs. The ultimate expressions for φ_1 and φ_2 are provided by relations (3.13), and the components of the vector potential are of the form

$$\psi_y = \frac{\rho_{22}}{4\pi\delta} \left[\frac{r^2 - 2z^2}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{z^2}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right], \quad (3.16)$$

$$\psi_z = \frac{\rho_{22}}{4\pi\delta} \left[\frac{3yz}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{yz}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right].$$

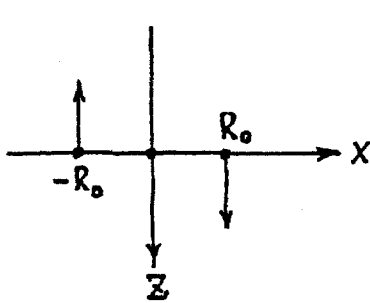


FIG. 1

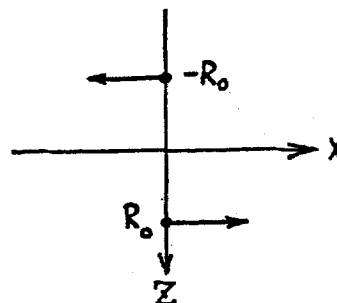


FIG. 2

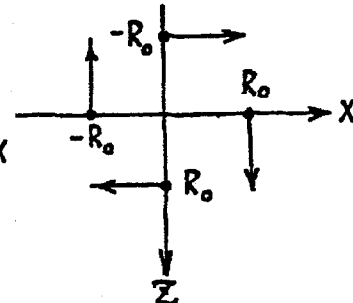


FIG. 3

3.6. Let the Biot medium be excited by the sources (3.12) and (3.15), and let the sign of the function $f(t)$ in (3.15) be changed (see Fig. 3). As in other cases, we will determine the limiting wave field as $R_0 \rightarrow 0$. The expressions for the longitudinal potentials φ_1 and φ_2 vanish, and the components of the vector potential $\vec{\psi}$ are as follows:

$$\begin{aligned}\psi_x &= -\frac{\rho_{22}}{4\pi\delta} \left[\frac{3xy}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{xy}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right], \\ \psi_y &= \frac{\rho_{22}}{4\pi\delta} \left[\frac{x^2 - 2y^2 + z^2}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{x^2 + z^2}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right], \\ \psi_z &= -\frac{\rho_{22}}{4\pi\delta} \left[\frac{3yz}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{yz}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right].\end{aligned}\quad (3.17)$$

Formulas (3.17) can be simplified by making use of the fact that the wave field is not affected by adding the gradient of an arbitrary function to the vector potential. If we add the gradient of the function

$$\phi = -\frac{\rho_{22}}{4\pi\delta} \frac{y}{R^3} \int_{R/v_3}^t \tau f(t-\tau) d\tau, \quad (3.18)$$

then formulas (3.17) are replaced by the relations

$$\psi_x = \psi_z = 0, \quad \psi_y = \frac{1}{v_3^2 R} f\left(t - \frac{R}{v_3}\right), \quad (3.19)$$

describing the wave field excited by a center of rotation about the y axis.

4. SOURCES REPRESENTED AS DISCONTINUITIES OF DISPLACEMENTS

4.1. Along with sources of the type of concentrated forces, we also consider sources related to discontinuities of displacements in both phases of Biot's medium. Let, in Biot's medium, the source given by the boundary conditions

$$\begin{aligned}[u_z^{(2)}] &= \frac{f(t)\delta(r)}{2\pi r}, \quad [\sigma_{zz}^{(2)}] = [\sigma_{rz}^{(2)}] = [\sigma_{\theta z}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_z^{(1)}] &= [u_r^{(2)}] = [u_\theta^{(2)}] = 0\end{aligned}\quad (4.1)$$

on the plane $z = 0$ be acting. This source is represented by a discontinuity of the displacement $u_z^{(2)}$ along the z axis. In this case, the expressions for the potentials are as follows:

$$\begin{aligned}\varphi_1 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (X_1 e^{-k|z|\alpha_1} + X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \varphi_2 &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} (A_1 X_1 e^{-k|z|\alpha_1} + A_2 X_2 e^{-k|z|\alpha_2}) e^{kt\eta} d\eta, \\ \psi_\theta &= \text{sign } z \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y e^{-k|z|\beta} e^{kt\eta} d\eta.\end{aligned}\quad (4.2)$$

Here, the functions X_1 , X_2 , and Y are defined by the equalities

$$X_1 = \frac{v_3^2(\rho_{22}g + 2A_2\rho_{12})F(k\eta)}{4\pi(A_2 - A_1)\rho_{22}\alpha_1\eta^2}, \quad X_2 = -\frac{v_3^2(\rho_{22}g + 2A_1\rho_{12})F(k\eta)}{4\pi(A_2 - A_1)\rho_{22}\alpha_2\eta^2}, \quad Y = -\frac{v_3^2 F(k\eta)}{2\pi\eta^2}, \quad g = 2 + \frac{\eta^2}{v_3^2}, \quad (4.3)$$

where the notation (2.6)–(2.8) is used.

In order to represent the potentials (4.2), corresponding to the source (4.1), in Stokes form, it is necessary to use the additional relations

$$\begin{aligned} \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{e^{k(t\eta-|z|\alpha)}}{\alpha\eta^2} F(k\eta) d\eta &= \frac{2z^2 - r^2}{R^5} \int_{R/v}^t \tau f(t-\tau) d\tau - \frac{r^2}{v^2 R^3} f\left(t - \frac{R}{v}\right), \\ \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{e^{k(t\eta-|z|\alpha)}}{\eta^2} F(k\eta) d\eta &= \frac{3r|z|}{R^5} \int_{R/v}^t \tau f(t-\tau) d\tau + \frac{r|z|}{v^2 R^3} f\left(t - \frac{R}{v}\right), \end{aligned} \quad (4.4)$$

which result from applying the operators $\partial/\partial\tau + 1/\tau$ and $\partial/\partial z$ to both sides of the second equality in (2.12). Using (2.10), (2.12), and (4.4), from (4.2) and (4.3) we derive the relations

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \frac{1}{4\pi(A_2 - A_1)R} \left\{ \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[f\left(t - \frac{R}{v_1}\right) - \frac{2v_3^2(A_2\rho_{12} + \rho_{22})r^2}{\rho_{22}R^2v_1^2} f\left(t - \frac{R}{v_1}\right) \right. \right. \\ &\quad \left. \left. - \frac{2v_3^2(A_2\rho_{12} + \rho_{22})(r^2 - 2z^2)}{\rho_{22}R^4} \int_{R/v_1}^t \tau f(t-\tau) d\tau \right] - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[f\left(t - \frac{R}{v_2}\right) \right. \right. \\ &\quad \left. \left. - \frac{2v_3^2(A_1\rho_{12} + \rho_{22})r^2}{\rho_{22}R^2v_2^2} f\left(t - \frac{R}{v_2}\right) - \frac{2v_3^2(A_1\rho_{12} + \rho_{22})(r^2 - 2z^2)}{\rho_{22}R^4} \int_{R/v_2}^t \tau f(t-\tau) d\tau \right] \right\}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \psi_x &= \frac{yz}{2\pi R^3} \left[f\left(t - \frac{R}{v_3}\right) + \frac{3v_3^2}{R^2} \int_{R/v_3}^t \tau f(t-\tau) d\tau \right], \\ \psi_y &= -\frac{xz}{2\pi R^3} \left[f\left(t - \frac{R}{v_3}\right) + \frac{3v_3^2}{R^2} \int_{R/v_3}^t \tau f(t-\tau) d\tau \right], \end{aligned}$$

which provide the desired representations of the potentials in Stokes form.

According to [10], the wave field (4.5) can be represented as a superposition of the wave fields excited by the following three sources: first, a force couple (3.1) acting along the z axis; second, a center of pressure (3.4) applied to the elastic phase, and, third, a center of pressure (2.16) acting on the fluid phase. All three sources are located at the origin of coordinates and are specified by the same time function $f(t)$ with the coefficients $2L$, A , and M , respectively. In proving this assertion, it is expedient to use the relations [4]

$$\frac{\rho_{22} + \rho_{12}A_{2,1}}{v_{1,2}^2\delta} = \frac{MA_{2,1} + \tilde{R}}{P\tilde{R} - M^2}, \quad \frac{\rho_{11}A_{2,1} + \rho_{12}}{v_{1,2}^2\delta} = \frac{PA_{2,1} + M}{P\tilde{R} - M^2}. \quad (4.6)$$

In the special case where conditions (1.5) are fulfilled in Biot's medium and, therefore, waves in the two phases are not correlated, we have $\varphi_1 = 0$, and the potential φ_2 (as follows from (4.2), (4.3), and (4.5)) is

determined by the formulas

$$\begin{aligned}\varphi_2 &= -\frac{1}{4\pi R} \left(1 - \frac{2v_3^2 r^2}{v_1^2 R^2}\right) f\left(t - \frac{R}{v_1}\right) + \frac{v_3^2(r^2 - 2z^2)}{2\pi R^5} \int_{R/v_1}^t \tau f(t - \tau) d\tau, \\ \varphi_2 &= -\frac{v_3^2}{4\pi} \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{gF(k\eta)}{\alpha_1 \eta^2} e^{k(t\eta - |z|\alpha_1)} d\eta.\end{aligned}\quad (4.7)$$

As above, the components of the vector potential are determined by (4.2), (4.3), and (4.5). Formulas (4.7) and the expressions (4.2), (4.3), and (4.5) for the potentials ψ_θ , ψ_x , and ψ_y determine the wave field in the elastic medium in the case of the source (4.1).

4.2. The source corresponding to a discontinuity of displacements in the fluid phase is given by the boundary conditions

$$[u_z^{(1)}] = \frac{\delta(r)f(t)}{2\pi r}, \quad [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad [\sigma_{zz}^{(2)}] = [\sigma_{rz}^{(2)}] = [\sigma^{(1)}] = 0. \quad (4.8)$$

The potentials of the field excited can be represented either by the integral relations (4.2), where

$$X_1 = -\frac{A_2 F(k\eta)}{4\pi(A_2 - A_1)\alpha_1}, \quad X_2 = \frac{A_1 F(k\eta)}{4\pi(A_2 - A_1)\alpha_2}, \quad Y = 0, \quad (4.9)$$

or by the equalities

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = -\frac{1}{4\pi(A_2 - A_1)R} \left[A_2 \begin{pmatrix} 1 \\ A_1 \end{pmatrix} f\left(t - \frac{R}{v_1}\right) - A_1 \begin{pmatrix} 1 \\ A_2 \end{pmatrix} f\left(t - \frac{R}{v_2}\right) \right]. \quad (4.10)$$

According to [10], the wave field of the source (4.8) can be represented as a superposition of the wave fields excited by a center of pressure (3.4), acting on the elastic phase, and a center of pressure (2.16), applied to the fluid phase. Both sources are located at the origin of coordinates and are specified by the time function $f(t)$ with the coefficients M and $\bar{R}$, respectively. In the special case where conditions (1.5) are fulfilled in Biot's medium, we have $\varphi_2 = 0$, and the potential φ_1 is determined by the relations

$$\varphi_1 = -\frac{1}{4\pi R} f\left(t - \frac{R}{v_1}\right) = -\frac{1}{4\pi} \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta)}{\alpha_2} e^{k(t\eta - |z|\alpha_2)} d\eta. \quad (4.11)$$

4.3. In Biot's medium, consider the source given by the conditions

$$\begin{aligned}[u_x^{(2)}] &= \frac{\delta(r)f(t)}{2\pi r}, \quad [u_y^{(2)}] = 0, \quad [\sigma_{zz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_z^{(2)}] &= [u_z^{(1)}] = 0, \quad [\sigma_{zx}^{(2)}] = [\sigma_{zy}^{(2)}] = 0\end{aligned}\quad (4.12)$$

and related to a discontinuity of the tangential displacement in the elastic phase. In determining the corresponding wave field, it is expedient to use the results in [11, 12], where the action of tangential forces in an elastic medium was studied and to take into account the evenness or oddness with respect to z of all the resulting potentials, displacements, and stresses. Since $u_z^{(2)}$, $u_z^{(1)}$, $\sigma_{zx}^{(2)}$, and $\sigma_{zy}^{(2)}$ are even and continuous, the last four conditions in (4.12) are automatically fulfilled. Thus, we only need to satisfy the first four conditions in (4.12). To this end, we will use the relations

$$\begin{aligned}u_r^{(2)} &= \frac{\partial \varphi_2}{\partial r} + \frac{1}{r} \frac{\partial \psi_z}{\partial \theta} - \cos \theta \frac{\partial \psi_y}{\partial z}, \\ u_\theta^{(2)} &= \frac{1}{r} \frac{\partial \varphi_2}{\partial \theta} - \frac{\partial \psi_z}{\partial r} + \sin \theta \frac{\partial \psi_y}{\partial z}, \\ \sigma^{(1)} &= \rho_{12} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_1}{\partial t^2}, \\ \sigma_{zz}^{(2)} &= \rho_{11} \frac{\partial^2 \varphi_2}{\partial t^2} + \rho_{22} \frac{\partial \varphi_1}{\partial t^2} - 2L \left(\frac{\partial^2 \varphi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi_2}{\partial \theta^2} - \cos \theta \frac{\partial^2 \psi_y}{\partial r \partial z} + \frac{\sin \theta}{r} \frac{\partial^2 \psi_y}{\partial \theta \partial z} \right),\end{aligned}\quad (4.13)$$

which enable one to express the displacements and stresses involved in the first four conditions (4.12) in terms of the potentials. Based on the boundary conditions, for the potentials we can write the representations

$$\begin{aligned}
\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= -\operatorname{sign} z \cos \theta \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} X_1 e^{-k|z|\alpha_1} + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} X_2 e^{-k|z|\alpha_2} \right] e^{kt\eta} d\eta, \\
\psi_y &= \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y e^{-k|z|\beta} e^{kt\eta} d\eta, \\
\psi_z &= -\operatorname{sign} z \sin \theta \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \beta Z e^{kt\eta - k|z|\beta} d\eta.
\end{aligned} \tag{4.14}$$

Here, the functions X_1 , X_2 , Y , and Z are defined as follows:

$$X_1 = -\frac{L(\rho_{12}A_2 + \rho_{22})F(k\eta)}{2\pi\delta(A_2 - A_1)\eta_2}, \quad X_2 = \frac{L(\rho_{12}A_1 + \rho_{22})F(k\eta)}{2\pi\delta(A_2 - A_1)\eta^2}, \quad Z = \frac{v_3^2 F(k\eta)}{2\pi\beta\eta^2}, \quad Y = \frac{v_3^2 g F(k\eta)}{4\pi\beta\eta^2}. \tag{4.15}$$

Using the integral relations (2.10) and (2.12), the expressions (4.14) with regard for (4.15) can be written in the form

$$\begin{aligned}
\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \frac{L \cos \theta}{2\pi\delta(A_2 - A_1)} \left\{ \begin{pmatrix} 1 \\ A_1 \end{pmatrix} (\rho_{12}A_2 + \rho_{22}) \left[\frac{3rz}{R^5} \int_{R/v_1}^t \tau f(t-\tau) d\tau + \frac{rz}{v_1^2 R^3} f\left(t - \frac{R}{v_1}\right) \right] \right. \\
&\quad \left. - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} (\rho_{12}A_1 + \rho_{22}) \left[\frac{3rz}{R^5} \int_{R/v_2}^t \tau f(t-\tau) d\tau + \frac{rz}{v_2^2 R^3} f\left(t - \frac{R}{v_2}\right) \right] \right\}, \\
\psi_y &= \frac{v_3^2}{2\pi} \left[\frac{2z^2 - r^2}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{z^2 - r^2}{2R^3 v_3^2} f\left(t - \frac{R}{v_3}\right) \right], \\
\psi_z &= -\frac{v_3^2}{2\pi} \left[\frac{3yz}{R^5} \int_{R/v_3}^t \tau f(t-\tau) d\tau + \frac{yz}{R^3 v_3^2} f\left(t - \frac{R}{v_3}\right) \right].
\end{aligned} \tag{4.16}$$

These representations, like relations (4.5) and (4.10), can be regarded as the Stokes formulas for Biot's medium.

In the special case where conditions (1.5) are fulfilled, we have $\varphi_1 = 0$, and the potential φ_2 is determined by either of the relations

$$\begin{aligned}
\varphi_2 &= -\frac{v_3^2 \cos \theta}{2\pi} \left[\frac{3rz}{R^5} \int_{R/v_1}^t \tau f(t-\tau) d\tau + \frac{rz}{v_1^2 R^3} f\left(t - \frac{R}{v_1}\right) \right], \\
\varphi_2 &= -\frac{v_3^2 \cos \theta \operatorname{sign} z}{2\pi} \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{F(k\eta)}{\eta^2} e^{kt\eta - k|z|\alpha_1} d\eta.
\end{aligned} \tag{4.17}$$

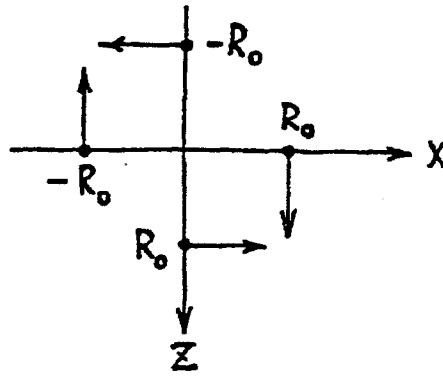


FIG. 4

As previously, the potentials ψ_y and ψ_z are described by (4.14) and (4.15).

The wave field (4.16) is also excited by the source involving two force couples (see Fig. 4) and given by conditions (3.12) and (3.15), in which the time function has the additional coefficient L . In order to verify this assertion, it is necessary to subtract the gradient of the function (3.18) from the vector potential.

Thus, all of the three sources related to jumps of displacements are equivalent to force sources. In these equivalent sources, the summarized forces and moments are zero, which is a consequence of the general laws of mechanics.

5. DISPLACEMENT FIELDS

In this section, we will determine the displacement fields corresponding to sources represented either in the form of a concentrated force or in the form of a point discontinuity of displacements. As a rule, the related formulas will be written both in terms of integral transforms and in Stokes form.

5.1. For the source corresponding to the concentrated force given by boundary conditions (2.1), the displacement field (in accordance with relations (2.4) and (2.5)) is described by the equalities

$$\begin{aligned} \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} &= - \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \alpha_1 X_1 e^{-k|z|\alpha_1} \right. \\ &\quad \left. + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \alpha_2 X_2 e^{-k|z|\alpha_2} + \begin{pmatrix} \rho_{12}/\rho_{22} \\ -1 \end{pmatrix} Y e^{-k|z|\beta} \right] e^{kt\eta} d\eta, \\ \begin{pmatrix} u_r^{(1)} \\ u_r^{(2)} \end{pmatrix} &= -\text{sign } z \int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} X_1 e^{-k|z|\alpha_1} \right. \\ &\quad \left. + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} X_2 e^{-k|z|\alpha_2} + \begin{pmatrix} \rho_{12}/\rho_{22} \\ -1 \end{pmatrix} Y e^{-k|z|\beta} \right] e^{kt\eta} d\eta, \end{aligned} \quad (5.1)$$

where the notation (2.6)–(2.8) is used. Using (4.4) and the relation

$$\int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{\alpha F(k\eta) e^{k(t\eta-|z|\alpha)}}{\eta^2} d\eta = \frac{2z^2 - r^2}{R^5} \int_{R/v}^\tau \tau f(t-\tau) d\tau + \frac{z^2}{v^2 R^3} f\left(t - \frac{R}{v}\right), \quad (5.2)$$

we can replace equalities (5.1) by the relations

$$\begin{aligned} \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} &= \frac{1}{4\pi\delta(A_2 - A_1)} \left\{ \frac{\partial^2}{\partial z^2} \begin{pmatrix} 1 \\ R \end{pmatrix} \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} + \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{(\rho_{12}A_2 + \rho_{22})z^2}{v_1^2 R^3} f\left(t - \frac{R}{v_1}\right) \right. \\ &\quad \left. - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{(\rho_{12}A_1 + \rho_{22})z^2}{v_2^2 R^3} f\left(t - \frac{R}{v_2}\right) + \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \frac{(A_2 - A_1)r^2}{v_3^2 R^3} f\left(t - \frac{R}{v_3}\right) \right\}, \end{aligned} \quad (5.3)$$

$$\begin{pmatrix} u_x^{(1)} \\ u_x^{(2)} \end{pmatrix} = \frac{1}{4\pi\delta(A_2 - A_1)} \left\{ \frac{\partial^2}{\partial x \partial z} \left(\frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} + \frac{xz}{R^3} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{(\rho_{12}A_2 + \rho_{22})}{v_1^2} f\left(t - \frac{R}{v_1}\right) - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\rho_{12}A_1 + \rho_{22}}{v_2^2} f\left(t - \frac{R}{v_2}\right) - \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \frac{(A_2 - A_1)}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right] \right\},$$

where

$$\begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} = \begin{pmatrix} 1 \\ A_1 \end{pmatrix} (\rho_{12}A_2 + \rho_{22}) \int_{R/v_1}^{R/v_3} \tau f(t - \tau) d\tau - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} (\rho_{12}A_1 + \rho_{22}) \int_{R/v_2}^{R/v_3} \tau f(t - \tau) d\tau. \quad (5.4)$$

In some cases, in relations (5.3) it is reasonable to perform the following change of variables:

$$\frac{\partial^2}{\partial z^2} \frac{1}{R} = \frac{2z^2 - \tau^2}{R^5}, \quad \frac{\partial^2}{\partial x \partial z} \frac{1}{R} = \frac{3xz}{R^5}. \quad (5.5)$$

In the special case where in the Biot medium conditions (1.5) are fulfilled, the displacements in the fluid phase are zero, whereas the displacements in the elastic phase are determined by the relations

$$\begin{aligned} u_z^{(2)} &= -\frac{1}{4\pi\rho_{11}R^3} \left[\frac{2z^2 - \tau^2}{R^2} \int_{R/v_1}^{R/v_3} \tau f(t - \tau) d\tau + \frac{z^2}{v_1^2} f\left(t - \frac{R}{v_2}\right) + \frac{\tau^2}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right], \\ u_x^{(2)} &= -\frac{xz}{4\pi\rho_{11}R^3} \left[\frac{3}{R^2} \int_{R/v_1}^{R/v_3} \tau f(t - \tau) d\tau + \frac{1}{v_1^2} f\left(t - \frac{R}{v_1}\right) - \frac{1}{v_3^2} f\left(t - \frac{R}{v_3}\right) \right]. \end{aligned} \quad (5.6)$$

Relations (5.6) coincide with the corresponding formulas in [6] up to sign. The latter distinction is inessential because it concerns only the sign of the force specifying the source.

5.2. Consider the source (4.1) related to a discontinuity of displacements in the elastic phase. In accordance with (2.4) and (4.2), the displacement field is described, in this case by the equalities

$$\begin{aligned} \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} &= -\text{sign } z \int_0^\infty \frac{k J_0(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \alpha_1 X_1 e^{-k|z|\alpha_1} + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \alpha_2 X_2 e^{-k|z|\alpha_2} + \begin{pmatrix} \rho_{12}/\rho_{22} \\ -1 \end{pmatrix} Y e^{-k|z|\beta} \right] e^{kt\eta} d\eta, \\ \begin{pmatrix} u_r^{(1)} \\ u_r^{(2)} \end{pmatrix} &= -\int_0^\infty \frac{k J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} X_1 e^{-k|z|\alpha_1} + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} X_2 e^{-k|z|\alpha_2} + \begin{pmatrix} \rho_{12}/\rho_{22} \\ -1 \end{pmatrix} Y e^{-k|z|\beta} \right] e^{kt\eta} d\eta. \end{aligned} \quad (5.7)$$

Here, the notation (4.3), (2.6), and (2.7) is used. Applying the formulas

$$\begin{aligned}
& \int_0^\infty \frac{k^2 J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F(k\eta) \frac{e^{k(t\eta-|z|\alpha)}}{\alpha \eta^2} d\eta = -\frac{r^3}{v^3 R^4} f' \left(t - \frac{R}{v} \right) \\
& + 3r \frac{4z^2 - r^2}{R^7} \int_{R/v}^t \tau f(t-\tau) d\tau + 2r \frac{2z^2 - r^2}{v^2 R^5} f \left(t - \frac{R}{v} \right), \\
& \int_0^\infty \frac{k^2 J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F(k\eta) \frac{\alpha e^{k(t\eta-|z|\alpha)}}{\eta^2} d\eta = -\frac{r z^2}{v^3 R^4} f' \left(t - \frac{R}{v} \right) \\
& + 3r \frac{4z^2 - r^2}{R^7} \int_{R/v}^t \tau f(t-\tau) d\tau + r \frac{5z^2 - r^2}{v^2 R^5} f \left(t - \frac{R}{v} \right), \\
& \int_0^\infty \frac{k^2 J_1(kr) dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F(k\eta) \frac{e^{k(t\eta-|z|\alpha)}}{\alpha} d\eta = \frac{r}{R^3} \left[f \left(t - \frac{R}{v} \right) + \frac{R}{v} f' \left(t - \frac{R}{v} \right) \right],
\end{aligned} \tag{5.8}$$

we can reduce equalities (5.7) to the form

$$\begin{aligned}
& \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} = \frac{v_3^2}{2\pi \rho_{22}(A_2 - A_1)R^7} \left(\frac{\partial^3}{\partial z^3} \frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} \\
& - \frac{1}{4\pi(A_2 - A_1)} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{z}{R^3} f \left(t - \frac{R}{v_1} \right) + \frac{z}{v_1 R^2} f' \left(t - \frac{R}{v_1} \right) \right] \\
& + \frac{1}{4\pi(A_2 - A_1)} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{z}{R^3} f \left(t - \frac{R}{v_2} \right) + \frac{z}{v_2 R^2} f' \left(t - \frac{R}{v_2} \right) \right] \\
& - \frac{v_3^2(A_2 \rho_{12} + \rho_{22})z}{2\pi \rho_{22}(A_2 - A_1)v_1^2 R^4} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{2(z^2 - 2r^2)}{R} f \left(t - \frac{R}{v_1} \right) - \frac{r^2}{v_1} f' \left(t - \frac{R}{v_1} \right) \right] \\
& + \frac{v_3^2(A_1 \rho_{12} + \rho_{22})z}{2\pi \rho_{22}(A_2 - A_1)v_2^2 R^4} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{2(z^2 - 2r^2)}{R} f \left(t - \frac{R}{v_2} \right) - \frac{r^2}{v_2} f' \left(t - \frac{R}{v_2} \right) \right] \\
& + \frac{z}{2\pi \rho_{22} R^4} \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \left[\frac{2(z^2 - 2r^2)}{R} f \left(t - \frac{R}{v_3} \right) - \frac{r^2}{v_3} f' \left(t - \frac{R}{v_3} \right) \right], \\
& \begin{pmatrix} u_x^{(1)} \\ u_x^{(2)} \end{pmatrix} = \frac{v_3^2}{2\pi \rho_{22}(A_2 - A_1)R^7} \left(\frac{\partial^3}{\partial r \partial z^2} \frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} \\
& - \frac{1}{4\pi(A_2 - A_1)} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{x}{R^3} f \left(t - \frac{R}{v_1} \right) + \frac{x}{v_1 R^2} f' \left(t - \frac{R}{v_1} \right) \right] \\
& + \frac{1}{4\pi(A_2 - A_1)} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{x}{R^3} f \left(t - \frac{R}{v_2} \right) + \frac{x}{v_2 R^2} f' \left(t - \frac{R}{v_2} \right) \right] \\
& + \frac{v_3^2(A_2 \rho_{12} + \rho_{22})x}{2\pi \rho_{22}(A_2 - A_1)v_1^2 R^4} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{2(r^2 - 2z^2)}{R} f \left(t - \frac{R}{v_1} \right) + \frac{r^2}{v_1} f' \left(t - \frac{R}{v_1} \right) \right] \\
& - \frac{v_3^2(A_1 \rho_{12} + \rho_{22})x}{2\pi \rho_{22}(A_2 - A_1)v_2^2 R^4} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{2(r^2 - 2z^2)}{R} f \left(t - \frac{R}{v_2} \right) + \frac{r^2}{v_2} f' \left(t - \frac{R}{v_2} \right) \right]
\end{aligned} \tag{5.9}$$

$$-\frac{x}{2\pi\rho_{22}R^4}\begin{pmatrix}\rho_{12} \\ -\rho_{22}\end{pmatrix}\left[\frac{r^2-5z^2}{R}f\left(t-\frac{R}{v_3}\right)-\frac{z^2}{v_3}f'\left(t-\frac{R}{v_3}\right)\right].$$

In (5.9), the notation (5.4) is used. In some cases, in these relations it is expedient to perform the change of variables in accordance with the formulas

$$\frac{\partial^3}{\partial r \partial z^2} \frac{1}{R} = 3r \frac{r^2 - 4z^2}{R^7}, \quad \frac{\partial^3}{\partial z^3} \frac{1}{R} = 3z \frac{3r^2 - 2z^2}{R^7}. \quad (5.10)$$

5.3. In the case of a concentrated discontinuity of displacements in the fluid phase of the form (4.8), the displacement field is determined by equalities (5.5), where the notation (4.9) is used. These relations imply the representations in Stokes form

$$\begin{aligned} \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} &= \frac{A_2 z}{4\pi(A_2 - A_1)R^2} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{1}{R} f\left(t - \frac{R}{v_1}\right) + \frac{1}{v_1} f'\left(t - \frac{R}{v_1}\right) \right] \\ &\quad - \frac{A_1 z}{4\pi(A_2 - A_1)R^2} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{1}{R} f\left(t - \frac{R}{v_2}\right) + \frac{1}{v_2} f'\left(t - \frac{R}{v_2}\right) \right], \\ \begin{pmatrix} u_x^{(1)} \\ u_x^{(2)} \end{pmatrix} &= \frac{A_2 x}{4\pi(A_2 - A_1)R^2} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{1}{R} f\left(t - \frac{R}{v_1}\right) + \frac{1}{v_1} f'\left(t - \frac{R}{v_1}\right) \right] \\ &\quad - \frac{A_1 x}{4\pi(A_2 - A_1)R^2} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{1}{R} f\left(t - \frac{R}{v_2}\right) + \frac{1}{v_2} f'\left(t - \frac{R}{v_2}\right) \right]. \end{aligned} \quad (5.11)$$

From equalities (4.10) and (5.11) it follows that the corresponding wave field is spherically symmetric.

5.4. In the case where the discontinuity of the tangential displacement $u_x^{(2)}$ is concentrated along the z axis (see (4.12)), we confine ourselves to presenting only the Stokes formulas

$$\begin{aligned} \begin{pmatrix} u_x^{(1)} \\ u_x^{(2)} \end{pmatrix} &= \frac{1}{2\pi\rho_{22}} \left\{ \frac{v_3^2}{A_2 - A_1} \left(\frac{\partial^3}{\partial x^2 \partial z} \frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} \right. \\ &\quad + \frac{v_3^2(\rho_{12}A_2 + \rho_{22})z}{(A_2 - A_1)v_1^2 R^4} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{y^2 + z^2 - 5x^2}{R} f\left(t - \frac{R}{v_1}\right) - \frac{x^2}{v_1} f'\left(t - \frac{R}{v_1}\right) \right] \\ &\quad - \frac{v_3^2(\rho_{12}A_1 + \rho_{22})z}{(A_2 - A_1)v_2^2 R^4} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{y^2 + z^2 - 5x^2}{R} f\left(t - \frac{R}{v_2}\right) - \frac{x^2}{v_2} f'\left(t - \frac{R}{v_2}\right) \right] \\ &\quad \left. + \frac{z}{2R^4} \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \left[3 \frac{3x^2 - y^2 - z^2}{R} f\left(t - \frac{R}{v_3}\right) + \frac{x^2 - y^2 - z^2}{v_3} f'\left(t - \frac{R}{v_3}\right) \right] \right\}, \end{aligned} \quad (5.12)$$

$$\begin{aligned} \begin{pmatrix} u_y^{(1)} \\ u_y^{(2)} \end{pmatrix} &= \frac{1}{2\pi\rho_{22}} \left\{ \frac{v_3^2}{A_2 - A_1} \left(\frac{\partial^3}{\partial x \partial y \partial z} \frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} \right. \\ &\quad - \frac{v_3^2(\rho_{12}A_2 + \rho_{22})xyz}{(A_2 - A_1)v_1^2 R^4} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{6}{R} f\left(t - \frac{R}{v_1}\right) + \frac{1}{v_1} f'\left(t - \frac{R}{v_1}\right) \right] \\ &\quad + \frac{v_3^2(\rho_{12}A_1 + \rho_{22})xyz}{(A_2 - A_1)v_2^2 R^4} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{6}{R} f\left(t - \frac{R}{v_2}\right) + \frac{1}{v_2} f'\left(t - \frac{R}{v_2}\right) \right] \\ &\quad \left. + \frac{xyz}{R^4} \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \left[\frac{6}{R} f\left(t - \frac{R}{v_3}\right) + \frac{1}{v_3} f'\left(t - \frac{R}{v_3}\right) \right] \right\}, \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} u_z^{(1)} \\ u_z^{(2)} \end{pmatrix} &= \frac{1}{2\pi\rho_{22}} \left\{ \frac{v_3^2}{A_2 - A_1} \left(\frac{\partial^3}{\partial x \partial z^2} \frac{1}{R} \right) \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} \right. \\ &\quad + \frac{v_3^2(\rho_{12}A_1 + \rho_{22})x}{(A_2 - A_1)v_1^2 R^4} \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{r^2 - 5z^2}{R} f\left(t - \frac{R}{v_1}\right) - \frac{z^2}{v_1} f'\left(t - \frac{R}{v_1}\right) \right] \end{aligned}$$

$$-\frac{v_3^2(\rho_{12}A_2 + \rho_{22})x}{(A_2 - A_1)v_2^2 R^4} \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{r^2 - 5z^2}{R} f\left(t - \frac{R}{v_2}\right) - \frac{z^2}{v_2} f'\left(t - \frac{R}{v_2}\right) \right] \\ + \frac{x}{R^4} \begin{pmatrix} \rho_{12} \\ -\rho_{22} \end{pmatrix} \left[3 \frac{3z^2 - r^2}{R} f\left(t - \frac{R}{v_3}\right) + \frac{z^2 - r^2}{v_3} f'\left(t - \frac{R}{v_3}\right) \right] \Bigg\}.$$

Here, the notation (5.4) is used. In some cases, the change of variables

$$\frac{\partial^3}{\partial x^2 \partial z} \frac{1}{R} = 3z \frac{y^2 + z^2 - 4x^2}{R^7}, \quad \frac{\partial^3}{\partial x \partial y \partial z} \frac{1}{R} = -\frac{15xyz}{R^7}, \quad \frac{\partial^2}{\partial x \partial z^2} \frac{1}{R} = 3x \frac{r^2 - 4z^2}{R^7} \quad (5.13)$$

is useful. From (5.12) it follows that the expressions for $u_x^{(1)}$ and $u_x^{(2)}$ differ from those for $u_z^{(1)}$ and $u_z^{(2)}$ only in that x and z are interchanged. The same conclusion can also be drawn by considering the sources (3.12) and (3.15), which are equivalent in action, as $R_0 \rightarrow 0$, to the source (4.12) up to the multiplier L . The wave field excited by two force couples (3.12) and (3.15) and represented in Fig. 4 is symmetric about the plane $x = z$. This symmetry implies the above-mentioned property of the wave field.

6. RADIATION DIAGRAMS

In this section, displacement fields are investigated in the far zone, where the spherical radius R is so large that, in expressions (5.3), (5.9), (5.11), and (5.12) for the displacements, only the terms decreasing as $1/R$, as R increases, are kept. In this zone, the wave field can be decomposed into individual waves $P1$, $P2$, and S , the first two of which are strictly longitudinal, whereas the third one is a strictly transverse wave. Indeed, on the fronts of the waves $P1$ and $P2$, the displacement vectors are directed either in the same sense as the ray or in the opposite sense. On the front of the wave S , the displacement vector is perpendicular to the ray. Here, we assume that the time function $f(t)$ of the source vanishes for $t < 0$ and satisfies either the condition $f(0) \neq 0$ in the case of the source (2.1) or the condition $f'(0) \neq 0$ in the case of the sources (4.1), (4.8), and (4.12).

Since sources act differently in different directions, it is useful to construct radiation diagrams for each of the sources. These diagrams are the plots of the dependences of displacements on wave fronts upon the angle in the polar coordinate system on the xz plane. The sources are located at the origin of coordinates. In Figs. 5–8, relative displacements on the fronts are plotted for sources of the following types: a vertical force (2.1), a discontinuity of the normal displacements in the skeleton (4.1), a discontinuity of the normal displacements in the fluid (4.8), and a jump of the tangential displacements in the elastic phase (4.12). In every figure, two diagrams marked by the symbols a and b and related to the fluid and solid phases, respectively, are presented. The arrows show the directions of displacements. The diagrams of the longitudinal waves $P1$ and $P2$ and of the transverse wave S are plotted by thin, semibold, and bold lines, respectively. For definiteness, it should be indicated that all of the radiation diagrams have been constructed for isotropic Biot medium with the following values of the parameters:

$$A = 4.43 \text{ GPa}, \quad L = 2.77 \text{ GPa}, \quad Q = 0.74 \text{ GPa}, \quad \bar{R} = 0.6 \text{ GPa}, \\ \alpha = 1.75, \quad \varepsilon = 0.26, \quad \rho_s = 2650 \text{ kg/m}^3, \quad \rho_f = 1000 \text{ kg/m}^3. \quad (6.1)$$

The velocities of propagation and the coefficients A_1 , A_2 , and $-\rho_{22}/\rho_{12}$, which determine the ratios of the amplitudes of the waves $P1$, $P2$, and S in the skeleton to those in the fluid phase, evaluated using (2.8), (1.3), and (2.7), are as follows:

$$v_1 = 2330 \text{ m/s}, \quad v_2 = 1030 \text{ m/s}, \quad v_3 = 1160 \text{ m/s}, \quad A_1 = 1.04, \quad A_2 = -0.12, \quad -\rho_{22}/\rho_{12} = +2.33. \quad (6.2)$$

The opposite directions of displacements in the two phases for the second longitudinal wave $P2$ is the distinctive feature of all of the diagrams. In addition, for this wave, the displacements in the fluid phase are an order of magnitude greater than those in the skeleton. For this reason, the largest relative displacement of particles almost always occurs in the second longitudinal wave. Apparently, the seismoelectric effect, which arises for large relative displacements of the fluid and the skeleton, is related to the second longitudinal

wave $P2$ [13]. It is also worth indicating that the displacements in the wave S in the fluid phase are quite considerable and only 2.3 times less than those in the skeleton.

The diagrams for the waves $P1$ and S in Fig. 5b are very similar to those for the waves P and S excited by a vertical force in an elastic medium [6], but the directions of the displacements in these two cases prove to be opposite. This difference is due to the fact that, in this paper, the applied force is given by the jump of the stress $\sigma_{zz}^{(2)}$, whereas in [14] the source is defined by a body force. The difference indicated conforms to the results in [10], where the body forces equivalent to jumps of displacements and stresses were determined.

The jump of $u_z^{(2)}$ treated as a source is characterized by the diagrams shown in Figs. 6. The diagram for the wave $P1$ is a superposition of the diagrams of a center of pressure and of a momentless couple of vertical forces [10]. The diagram of $P1$ looks like the diagram of the wave P corresponding to a jump of u_z in an elastic one-phase medium. For the wave $P2$, the displacements corresponding to the force couple are larger in absolute value than those corresponding to centers of pressure, and the displacements corresponding to these sources have opposite directions. These facts explain the four-petal pattern of the diagram, in which the displacements vanish along four directions. The diagrams of the wave S are the typical diagrams corresponding to a momentless couple of vertical forces [14]. For clarity, in Fig. 6b the displacements in the wave $P2$ are magnified two times.

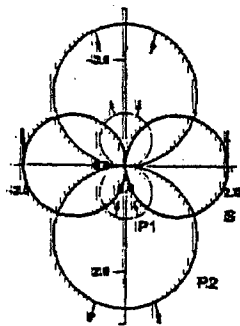


FIG. 5A

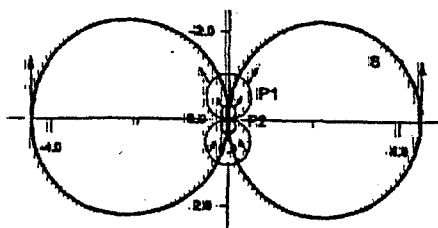


FIG. 5B

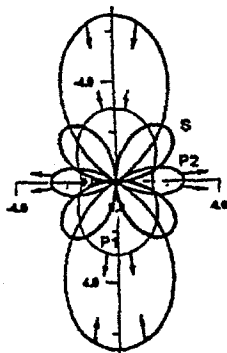


FIG. 6A

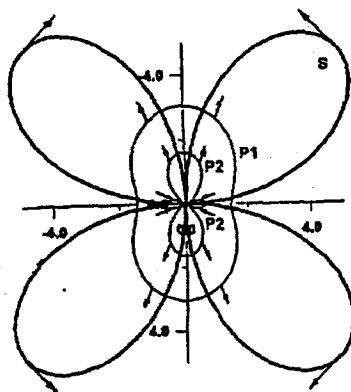


FIG. 6B

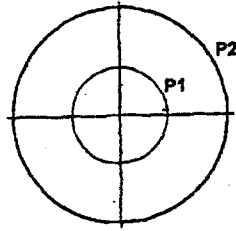


FIG. 7

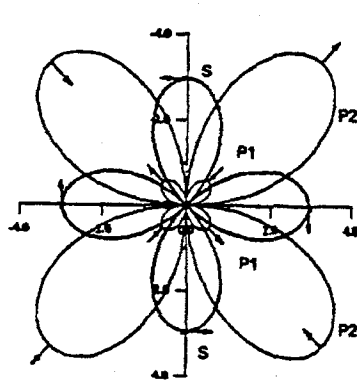


FIG. 8A

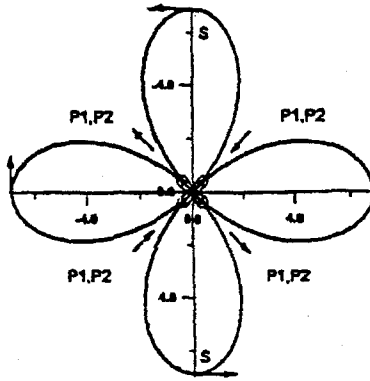


FIG. 8B

The jump of $u_z^{(1)}$ excites spherically symmetric longitudinal fields (see Fig. 7) in both phases. If we assume that the displacement in the fluid phase in the wave $P1$ is of unit length, then the displacement in the wave $P2$ in the same phase is equal to 19. The displacements of the skeleton in the waves $P1$ and $P2$ are then equal to 1.8 and 2.75, respectively.

All of the displacements are directed away from the center of the source, with the exception of the skeleton displacement in the wave $P2$, which is directed toward the center. Thus, among the waves excited by the source (4.8), the wave $P2$ is the most intensive one.

The jump of $u_x^{(2)}$, for which the equivalent force is represented by a double momentless force couple in the plane xz , is characterized by the four-petal diagrams of the waves $P1$, $P2$, and S (see Fig. 8), differing by an angle of 45° . Qualitatively, these diagrams are very similar to the diagrams corresponding to a double force couple in an elastic isotropic medium [6].

This work was supported by the Russian Foundation for Basic Research under grant No. 96-05-65904.

Translated by L. A. Molotkov.

REFERENCES

1. R. Burridge and C. A. Vargas, "The fundamental solution in dynamic poroelasticity," *Geophys. J. Roy. Astr. Soc.*, **58**, 61-90 (1979).
2. A. N. Norris, "Radiation from a point source and scattering theory in a fluid-saturated porous solid," *J. Acoust. Soc. Am.*, **77**, No. 6, 2012-2023 (1985).

3. G. Bonnet, "Basic singular solutions for a poroelastic medium in the dynamic range," *J. Acoust. Soc. Am.*, **82**, No. 5, 1758–1762 (1987).
4. L. A. Molotkov and A. V. Bakulin, "Sources of the center of dilatation and center of compression types in the Biot model," *Zap. Nauchn. Semin. POMI*, **230**, 196–213 (1995).
5. L. A. Molotkov, "On sources of the center of compression type in an isotropic and a transversely isotropic medium," in: *Vopr. Dinam. Teor. Raspr. Seismich. Voln*, Vol. 25, Leningrad Univ. Press (1986), pp. 76–85.
6. K. Aki and P. G. Richards, *Quantitative Seismology. Theory and Methods*, Freeman and Co., San Francisco (1980).
7. G. I. Petrashen, "Elements of the dynamic theory of propagation of seismic waves," in: *Vopr. Dinam. Teor. Raspr. Seismich. Voln*, Vol. 3, Leningrad Univ. Press (1950), pp. 11–106.
8. M. A. Biot, "Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range," *J. Acoust. Soc. Am.*, **28**, No. 2, 168–178 (1956).
9. H. Bateman and A. Erdelyi, *Tables of Integral Transforms*, Vol. 2, McGraw-Hill, New York (1954).
10. A. V. Bakulin, "Body forces equivalent to jumps of stresses and displacements in the Biot model of an anisotropic porous medium," this issue.
11. E. I. Shemyakin and V. L. Feinschmidt, "Wave propagation in an elastic half-space excited by a surface tangential force," *Uchen. Zap. Leningr. Univ.*, **179**, 148–179 (1954).
12. G. I. Petrashen and I. N. Uspenskii, "On wave propagation in layered-isotropic elastic media I," *Uchen. Zap. Leningr. Univ.*, **208**, 58–141 (1956).
13. N. M. Neustadt, Z. V. Mazanova, and N. D. Suvorova, *Seismo- and Piezoelectric Phenomena in Geophysics of Prospecting* [in Russian], Nedra, St. Petersburg (1992).
14. J. E. White, *Underground Sound: Application of Seismic Waves*, Elsevier, Amsterdam (1993).