# Advances in Anisotropy— Selected Theory, Modeling, and Case Studies

*Edited by Julie A. Hood*

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## Contents

# An effective media model for alternating layers of fluids and solids: A special case of the Biot model

Lev A. Molotkov[1] and Andrey V. Bakulin[2]

## ABSTRACT

By a comparison of both equations and wave fronts, we show that effective models of stratified fluid-solid media can be considered as special cases of the Biot model. Explicit relationships which transform all Biot parameters to the effective media domain are presented. In the case of isotropic or transversely isotropic elastic layers this is just a special case of the transversely isotropic Biot model. However, for the lower symmetry cases more complex Biot models are necessary, and are presented.

A model of alternating layers of solids with fluids can be used as a first approximation in representing fractured and porous media. However, use of traditional long-wave equivalent effective media models is not valid. Although this medium is not a strict porous medium (thus enabling the use of Biot theory formulations based on discontinuous pore space), neither are formulations based on monophase elasticity appropriate; even though normal displacements and stresses are continuous across boundaries for both phase, the constitutive equations representing this model are actually two-phase.

The effective media model is a transitional model between a completely two-phase model in which all stresses and displacements are different in both phases, unlike in a monophase model where the stresses and displacements are coincident. This transitional character can be seen in the expression of the wave propagation, in particular as a second longitudinal wave propagating along the layers yet absent across lamination, and generating a triangular-shaped wave front. A relationship established between models confirms the longitudinal nature of this front.

Additionally, two new features of wave propagation in the anisotropic Biot model are presented. Double loops on the shear wave front with four cusps in one quadrant are now observed. Additionally, the transition from slow longitudinal to shear mode occurs indicating that at orthogonal directions on the same front polarization changes from pure longitudinal to pure shear. These features can be used as potential indicators for the presence of anisotropic porous reservoirs.

## INTRODUCTION

Effective media models representing finely layered elastic media with nonwelded contacts have been widely studied by the geophysics community (eg., Molotkov, 1979; Schoenberg, 1983a; Cheadle

[1] St. Petersburg Branch of Steklov Mathematical Institute, Russian Academy of Sciences, St. Petersburg 191011, Fontanka 27, Russia, e-mail: Molotkov@pdmi.ras.ru

[2] St. Petersburg State University, Department of Geophysics, St. Petersburg 199034, University embankment 7/9, Russia; now at Schlumberger Cambridge Research, High Cross, Madingley road, Cambridge, United Kingdom, CB3 OEL, e-mail: bakulin@cambridge.scr.slb.com

et al., 1991; Rathore et al., 1991; Hsu and Schoenberg, 1993; Hood and Mignogna, 1994). However, media composed of alternating fluid and solid layers has received little attention (Brekhovskikh, 1973; Molotkov, 1979; Schoenberg, 1983b) in comparison to its importance as representing a simple example for fractured and porous media. Although this model is too simplistic to completely depict reservoir conditions, in order to begin more realistic modeling, wave propagation studies in porous media must be performed. Periodic porous media have been considered using homogenization approaches (Burridge and Keller, 1981) and other averaging techniques (Pride et al., 1992) which deduce the Biot equations starting at the microlevel. However, these expressions are rather complicated for use in direct analysis. Additionally, these methods do not provide an example for any kind of porous media, including even the simple case of alternating layers of fluids and solids. Nevertheless, their averaging methods lead to the same results (equations and parameters) as our method.

The first extensive investigations of wave propagation in a stratified fluid-solid medium included establishment of constitutive equations of the two-phase effective media model by Molotkov (1979), while Schoenberg (1983b) provided formulae and analysis of slowness curves. Molotkov (1979) also noted (without derivation) that the constitutive equations would be similar to those of Biot (1962). Other studies (Schoenberg, 1984; Molotkov and Khilo, 1984; Molotkov, 1988) constructed wave fronts in such media from a point source. The physical modeling of wave propagation in alternating fluid and solid layers conducted by Plona et al. (1987) showed good agreement between measured and calculated velocities for all propagation modes; the predicted second wave, often referred to as the Biot "slow" wave, was clearly observed as pure longitudinal along the layers, yet was absent across lamination. Despite the similarities in representative equations and propagation modes, the close relationship between effective media and Biot model has not yet been established.

The goal of this paper is to show that the effective media model for a stratified fluid-solid medium is just a special case of the transversely isotropic Biot model, providing the solid layers are at most transversely isotropic in complexity. Effective media models for systems having less symmetric solid layers are special cases of the Biot model as well, but with a more complex formulation. The focus of this paper will be on the relationships between models for both constitutive equations and wave fronts. Multiphase models of stratified media of fluid and solid layers with slide contact on the interfaces were derived and investigated by Molotkov and Khilo (1984) and Molotkov (1994b). A special case of these models is the two-phase effective media model of a stratified solid-fluid medium. The Biot model was generalized to three phases: one case is for two elastic and one fluid phase (Leclaire and Cohen-Tenoudji, 1994), while the other has only one of each solid, liquid, and gaseous phase (Camarasa, 1994). Clearly there is an analogous relationship between these multiphase models.

## THE TRANSVERSELY ISOTROPIC BIOT MODEL

In the Biot model it is assumed that at any physical point there are two vectors of displacement and two tensors of stress corresponding to the fluid and the solid phases, hereafter referred to as the "first" and "second" phase, respectively. Two phases are necessary because in any small neighborhood there are skeleton and fluid; in addition, relative motion between the fluid and solid is possible because the pore space is interconnected. For both phases, displacements $u_x^{(1)}$, $u_y^{(1)}$, $u_z^{(1)}$ and stress $t^{(1)}$ can be averaged over the fluid phase, while displacements $u_x^{(2)}$, $u_y^{(2)}$, $u_z^{(2)}$ and stresses $t_{xx}^{(2)}$, $t_{yy}^{(2)}$, $t_{zz}^{(2)}$, $t_{yz}^{(2)}$, $t_{xz}^{(2)}$, $t_{xy}^{(2)}$ can be averaged over the solid phase. Superscripts $^{(1)}$ and $^{(2)}$ represent the separate fluid and solid phase components, respectively. These averages characterize the displacement and stress at any point of a porous medium.

Since every phase occupies a part of the total volume, it is necessary to average the stress not just over one phase but over the total bulk volume at any point in a neighborhood. Let porosity $\varepsilon$ represent the part of the volume occupied by fluid. Then the total volume averaged stresses corresponding to both solid and fluid phases are given by

$$\sigma_{ij}^{(2)} = (1-\varepsilon)t_{ij}^{(2)}, \; \sigma^{(1)} = \varepsilon t^{(1)} , \tag{1}$$

respectively, for $i, j = x, y, z$. Setting the symmetry axis along $z$, the equations of a transversely isotropic Biot medium in terms of the stresses averaged over the total volume and displacements averaged over their respective phase volume are, according to Hooke's law,

$$\begin{aligned}
\sigma_{xx}^{(2)} &= P\frac{\partial u_x^{(2)}}{\partial x} + A\frac{\partial u_y^{(2)}}{\partial y} + F\frac{\partial u_z^{(2)}}{\partial z} + M\theta \, , \\
\sigma_{yy}^{(2)} &= A\frac{\partial u_x^{(2)}}{\partial x} + P\frac{\partial u_y^{(2)}}{\partial y} + F\frac{\partial u_z^{(2)}}{\partial z} + M\theta \, , \\
\sigma_{zz}^{(2)} &= F\frac{\partial u_x^{(2)}}{\partial x} + F\frac{\partial u_y^{(2)}}{\partial y} + C\frac{\partial u_z^{(2)}}{\partial z} + Q\theta \, , \\
\sigma^{(1)} &= M\frac{\partial u_x^{(2)}}{\partial x} + M\frac{\partial u_y^{(2)}}{\partial y} + Q\frac{\partial u_z^{(2)}}{\partial z} + R\theta \, , \\
\sigma_{yz}^{(2)} &= L\left(\frac{\partial u_y^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial y}\right) \, , \quad \sigma_{xz}^{(2)} = L\left(\frac{\partial u_x^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial x}\right) \, , \\
\sigma_{xy}^{(2)} &= N\left(\frac{\partial u_x^{(2)}}{\partial y} + \frac{\partial u_y^{(2)}}{\partial x}\right) \, , \quad \theta = div\vec{u}^{(1)} = \frac{\partial u_x^{(1)}}{\partial x} + \frac{\partial u_y^{(1)}}{\partial y} + \frac{\partial u_z^{(1)}}{\partial z} \, ,
\end{aligned} \tag{2}$$

where the matrix of coefficients relating stress to strain from equation (2) is positive definite and $P = A + 2N$. Additionally, displacements and stresses satisfy the equilibrium equations

$$\begin{aligned}
\frac{\partial \sigma_{xx}^{(2)}}{\partial x} + \frac{\partial \sigma_{xy}^{(2)}}{\partial y} + \frac{\partial \sigma_{xz}^{(2)}}{\partial z} &= \rho'_{11}\ddot{u}_x^{(2)} + \rho'_{12}\ddot{u}_x^{(1)} \, , \\
\frac{\partial \sigma_{xy}^{(2)}}{\partial x} + \frac{\partial \sigma_{yy}^{(2)}}{\partial y} + \frac{\partial \sigma_{yz}^{(2)}}{\partial z} &= \rho'_{11}\ddot{u}_y^{(2)} + \rho'_{12}\ddot{u}_y^{(1)} \, , \\
\frac{\partial \sigma_{xz}^{(2)}}{\partial x} + \frac{\partial \sigma_{yz}^{(2)}}{\partial y} + \frac{\partial \sigma_{zz}^{(2)}}{\partial z} &= \rho''_{11}\ddot{u}_z^{(2)} + \rho''_{12}\ddot{u}_z^{(1)} \, , \\
\frac{\partial \sigma^{(1)}}{\partial x} &= \rho'_{12}\ddot{u}_x^{(2)} + \rho'_{22}\ddot{u}_x^{(1)} \, , \\
\frac{\partial \sigma^{(1)}}{\partial y} &= \rho'_{12}\ddot{u}_y^{(2)} + \rho'_{22}\ddot{u}_y^{(1)} \, , \\
\frac{\partial \sigma^{(1)}}{\partial z} &= \rho''_{12}\ddot{u}_z^{(2)} + \rho''_{22}\ddot{u}_z^{(1)} \, ,
\end{aligned} \tag{3}$$

where each dot above the vector components represents a single differentiation with respect to time. Although the original Biot formulation does not provide expressions for these matrix coefficients as functions of solid and fluid phase parameters and porosity, subsequent papers (eg., Biot and Willis, 1957) developed methods based on experimental determination. Averaging techniques by Burridge and Keller (1980) and Pride et al. (1992) rederived the Biot equations from fundamental solid and fluid equations at the microgeometry scale. Although explicit expressions for all parameters are formulated in those studies, their expressions are not readily applied and analyzed.

Another group of parameters is dependent on the density of both phases as well as the pore space geometry, and is defined as

$$\rho'_{11} = \rho_s(1-\varepsilon) + \varepsilon\rho_f(\alpha_1 - 1)\,, \qquad \rho''_{11} = \rho_s(1-\varepsilon) + \varepsilon\rho_f(\alpha_2 - 1)\,,$$

$$\rho'_{12} = -\rho_f\,\varepsilon(\alpha_1 - 1)\,, \qquad \rho''_{12} = -\rho_f\,\varepsilon(\alpha_2 - 1)\,,$$

$$\rho'_{22} = \rho_f\,\varepsilon\alpha_1\,, \qquad \rho''_{22} = \rho_f\,\varepsilon\alpha_2\,, \tag{4}$$

where $\rho_s$ and $\rho_f$ are the densities of solid and fluid material, respectively. The $\alpha_1$ and $\alpha_2$ are the components of the tortuosity tensor in the $x$ and $z$ directions, respectively, which describes the geometry of pore space. Transverse isotropy is assumed, which means that when symmetry is along the $z$ direction, tortuosities in the horizontal $xy$-plane are independent of direction and uncoupled from one another, i.e.

$$\alpha = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix}. \tag{5}$$

The transversely isotropic Biot model is therefore defined by 13 parameters: *A, F, M, C, Q, R, L, N,* $\rho_s$, $\rho_f$, $\alpha_1$, $\alpha_2$, and $\varepsilon$.

## EQUATIONS OF THE EFFECTIVE MEDIA MODEL OF A LAYERED FLUID-SOLID MEDIUM

Applying the methods of matrix averaging to stratified fluid-solid media, Molotkov (1979) obtained the constitutive equations for two-phase media as

$$\begin{aligned}
t_{xx}^{(2)} &= \lambda_0 b\Big[\frac{\partial u_z}{\partial z} + \varepsilon\Big(\frac{\partial u_x^{(1)}}{\partial x} + \frac{\partial u_y^{(1)}}{\partial y}\Big)\Big] + [a + (1-\varepsilon)\lambda_0 b^2]\frac{\partial u_x^{(2)}}{\partial x} + [2\mu b + (1-\varepsilon)\lambda_0 b^2]\frac{\partial u_y^{(2)}}{\partial y}\,,\\
t_{yy}^{(2)} &= \lambda_0 b\Big[\frac{\partial u_z}{\partial z} + \varepsilon\Big(\frac{\partial u_x^{(1)}}{\partial x} + \frac{\partial u_y^{(1)}}{\partial y}\Big)\Big] + [2\mu b + (1-\varepsilon)\lambda_0 b^2]\frac{\partial u_x^{(2)}}{\partial x} + [a + (1-\varepsilon)\lambda_0 b^2]\frac{\partial u_y^{(2)}}{\partial y}\,,\\
t_{zz} &= \lambda_0\Big[\frac{\partial u_z}{\partial z} + \varepsilon\Big(\frac{\partial u_x^{(1)}}{\partial x} + \frac{\partial u_y^{(1)}}{\partial y}\Big)\Big] + (1-\varepsilon)\lambda_0 b\Big(\frac{\partial u_x^{(2)}}{\partial x} + \frac{\partial u_y^{(2)}}{\partial y}\Big)\,,\\
t_{xy}^{(2)} &= \mu\Big(\frac{\partial u_x^{(2)}}{\partial y} + \frac{\partial u_y^{(2)}}{\partial x}\Big)\,,
\end{aligned} \tag{6}$$

with the equilibrium equations represented as

$$
\begin{gathered}
\frac{\partial t_{xx}^{(2)}}{\partial x}+\frac{\partial t_{xy}^{(2)}}{\partial y}=\rho_s\frac{\partial^2 u_x^{(2)}}{\partial t^2}\,,\qquad \frac{\partial t_{xy}^{(2)}}{\partial x}+\frac{\partial t_{yy}^{(2)}}{\partial y}=\rho_s\frac{\partial^2 u_y^{(2)}}{\partial t^2}\,,\\
\frac{\partial t^{(1)}}{\partial x}=\rho_f\frac{\partial^2 u_x^{(1)}}{\partial t^2}\,,\qquad \frac{\partial t^{(1)}}{\partial y}=\rho_f\frac{\partial^2 u_y^{(1)}}{\partial t^2}\,,\qquad \frac{\partial t_{zz}}{\partial z}=\bar{\rho}\frac{\partial^2 u_z}{\partial t^2}\,,
\end{gathered}
\tag{7}
$$

where

$$
\begin{gathered}
u_z=u_z^{(1)}=u_z^{(2)}\,,\quad t_{zz}=t_{zz}^{(2)}=t^{(1)}\,,\\
\frac{\partial u_z}{\partial z}=\varepsilon\frac{\partial u_z^{(1)}}{\partial z}+(1-\varepsilon)\frac{\partial u_z^{(2)}}{\partial z}\,,\\
\frac{\partial t_{zz}}{\partial z}=\varepsilon\frac{\partial t^{(1)}}{\partial z}+(1-\varepsilon)\frac{\partial t_{zz}^{(2)}}{\partial z}\,,
\end{gathered}
\tag{8}
$$

and

$$
\begin{gathered}
a=\frac{4\mu_2(\lambda_2+\mu_2)}{\lambda_2+2\mu_2}\,,\qquad b=\frac{\lambda_2}{\lambda_2+2\mu_2}\,,\\
\lambda_0=\left[\frac{\varepsilon}{\lambda_1}+\frac{1-\varepsilon}{\lambda_2+2\mu_2}\right]^{-1}\,,\qquad \bar{\rho}=\varepsilon\rho_f+(1-\varepsilon)\rho_s\,,
\end{gathered}
\tag{9}
$$

with $u_i^{(1)}$, $u_i^{(2)}$, $t^{(1)}$, $t_{ij}^{(2)}$ $(i, j = x, y, z)$ as previously defined. The $\lambda_2$, $\mu_2$ and $\rho_s$ are the Lamé parameters and density of the solid phase material, respectively; $\lambda_1$ and $\rho_f$ are the Lamé parameter and density of the fluid phase material, respectively; and $\varepsilon$ is porosity. These six parameters $\lambda_2$, $\mu_2$, $\rho_s$, $\lambda_1$, $\rho_f$, $\varepsilon$ completely define the medium.

## TRANSFORMATION OF BIOT MODEL TO THE EFFECTIVE MEDIA MODEL

Taking into account the relationships of equations (7) and (9), a direct comparison of the two sets of equations, (2) and (3) with (6) and (8), shows that the equations of the transversely isotropic Biot medium transform to the equations of the effective media model, providing that the thirteen Biot parameters are properly expressed through the six parameters of the effective media model.

It is instructive to make this transition step-by-step in order to note from the physical point of view which parameters and associated conditions control the characteristic wave propagation effects, such as the origins of triangular wave fronts which often occur in effective media models (Molotkov, 1988; Schoenberg, 1984). For this purpose, three intermediate models are considered; each successive model a more specialized case of the previous one. Equations governing each model are derived and wave fronts from a point source are presented. To construct the wave fronts in the Biot and effective media models, a parametric description of the wave fronts in the form $x = x(\tau)$ and $z = z(\tau)$ is used which is applicable to transversely isotropic elastic and porous media (Molotkov and Khilo, 1984). Analytic descriptions of the wave fronts provides a means to derive expressions for velocities along axes, investigate the curvature of fronts, and find angular points or cusps.

A more detailed comparison of wave fronts in a transversely isotropic Biot model with the effective media model of a stratified fluid-solid medium with isotropic layers can be found in (Molotkov and Bakulin, 1998); a medium composed of alternating layers of fluid and solid anisotropic elastic layers, however, was not considered in that paper.

The first step in the transition from a pure Biot model to an effective media model is to consider a model where shear modulus $L = 0$, indicating that the layered fluid-solid medium does not support shear stress in the *xz*-plane. The transformation from the Biot model to the first intermediate model is shown in Figures 1 and 2, where wave fronts (group velocities) from a point source in the *xz*-plane for the Biot model (Figure 1) and Model 1 (Figure 2) are represented for $L = 0.2$ GPa

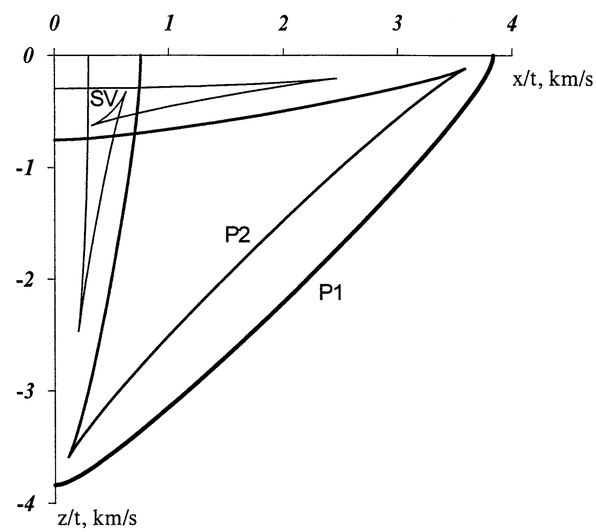


**FIG. 1. Wave fronts in a transversaly isotropic porous medium for small $L$ ($L = 0.2$ GPa with $P = C = 31.2$ GPa, $F = 4.3$ GPa, $M = Q = 1.1$ GPa, $R = 0.4$ GPa, $\rho_s = 2700$ kg/m³, $\rho_f = 1000$ kg/m³, $\alpha_1 = \alpha_2 = 3$, porosity = 20%) The bold line corresponds to the fast quasi-longitudinal wave *P*1, the semi-bold line represents the second or "slow" quasi-longitudinal wave *P*2, and the thin line is for the quasi-shear *SV* wave. Note the double loop on the *SV* wave front.**

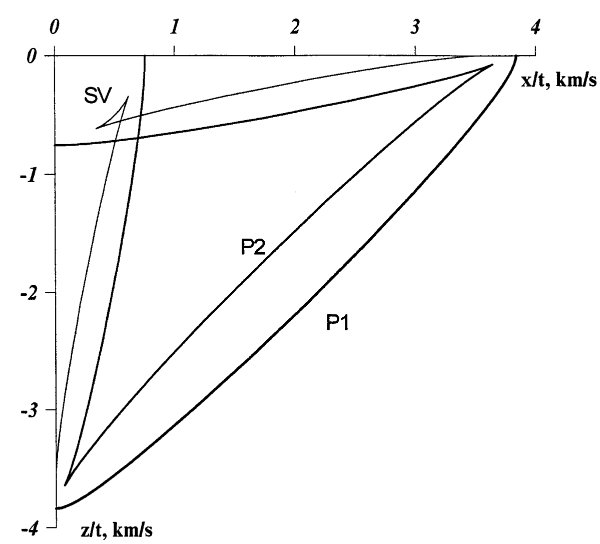


**FIG. 2. Wave fronts in a transversaly isotropic porous medium for Model 1 with $L = 0$ and all other parameters as in Fig. 1. Note the survival of the double loop from the previous figure. The bold line corresponds to the fast quasi-longitudinal wave *P*1, the semi-bold line represents the second or "slow" quasi-longitudinal wave *P*2, and the thin line is for the quasi-shear *SV* wave.**

and $L = 0$, respectively. On the *SV* wave front a double loop (or loop of second order) with four cusps is observed (Figure 1). As $L$ approaches zero, the convex parts of outer loop advance toward the coordinate axes and destructively interfere with the corresponding fronts from the adjacent quadrants to effectively vanish (Figure 2).

The next intermediate step is Model 2 which incorporates the additional constraint of proportionality between coefficients of the $\sigma_{zz}^{(2)}$ and $\sigma^{(1)}$ terms of equation (2):

$$\frac{F}{M} = \frac{C}{Q} = \frac{Q}{R} = \frac{(1-\varepsilon)}{\varepsilon}\ , \tag{10}$$

so that

$$\varepsilon\sigma_{zz}^{(2)} = (1-\varepsilon)\sigma^{(1)},\ t_{zz}^{(2)} = t^{(1)}. \tag{11}$$

Equation (11) is simply the boundary condition requiring that internal normal stresses along the $z$-axis are continuous across the fluid-solid interfaces. The relationships of equation (11) hold at all points throughout the medium due to the long wavelength criteria, i.e., when the thickness of the fluid and solid layers is sufficiently small compared with the measuring wavelength. As parameters $F, M, C, Q, R$ approach values satisfying the conditions of equation (10), the velocities of waves $P2$ and *SV* along the $z$-axis decrease to zero, whereas the upper angular point of the $P2$ front approaches the intersection of the *SV* front with the $x$-axis (Figure 3). When the conditions of equation (10) are strictly satisfied (as in Model 2), the parts of the $P2$ and *SV* fronts corresponding to the upper angular point of the $P2$ front continue towards the $x$-axis and cancel from destructive interference with fronts from the adjacent quadrants (Figure 4).

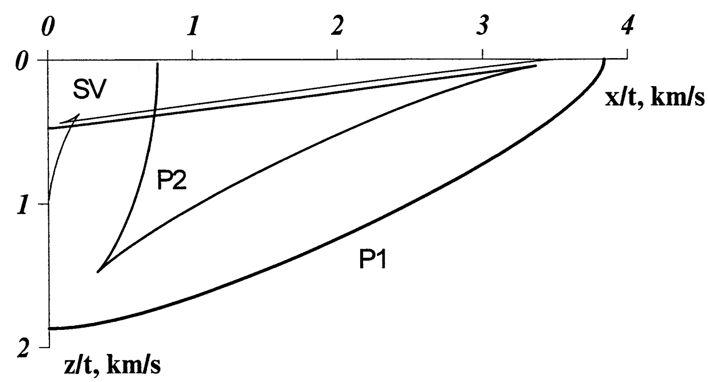


**FIG. 3. Wave fronts for the intermediate Biot model, where conditions of equation (10) are approximately satisfied, corresponding to preferential alignment of fracture planes (fluid layers) parallel to the $x$-axis. The bold line corresponds to the fast quasi-longitudinal wave $P1$, the semi-bold line represents the second or "slow" quasi-longitudinal wave $P2$, and the thin line is for the quasi-shear *SV* wave.**

The final step in the transition from a Biot model to an effective media model is represented by Model 3 which has the additional conditions

$$\alpha_1 = 1, \alpha_2 = \infty, \tag{12}$$

which correspond to the absence of mechanical coupling between solid and fluid displacements along the $x$-axis, and full coupling of both phases in orthogonal directions, respectively. The relationships of equation (12) were first suggested by Schoenberg and Sen (1983) who noted that wave propagation in a medium of alternating fluid-fluid layers could be described on the basis of the anisotropic Biot model. In order to keep kinetic energy in Model 3 finite, while $\alpha_2 \to \infty$ it is necessary to assume that

$$u_z^{(1)} = u_z^{(2)}. \tag{13}$$

This implies that normal displacements are continuous at every fluid-solid interface. When these interfaces are densely distributed, equation (13) holds at any point in the medium. This additional constraint does not change the shape of wavefronts shown in Figure 4 but does affect the velocity values along the coordinate axes.

Parameters $P$, $F$, and $C$ for Model 3 have expressions

$$P = (1 - \varepsilon)\,[a + (1 - \varepsilon)\,\lambda_0 b^2]\,,$$

$$F = (1 - \varepsilon)^2\,\lambda_0 b\,,$$

and

$$C = (1 - \varepsilon)^2\,\lambda_0\,, \tag{14}$$

where $\lambda_1$ is the Lamé parameter of the fluid phase; the $\lambda_2$ and $\mu_2$ are the Lamé parameters of the solid phase; and $a$, $b$, $\lambda_0$ are given by equation (9). Strictly satisfying the conditions of equation (14) transforms the Biot model into the equations of the effective media model for a stratified fluid-solid medium derived by Molotkov (1979; 1991).

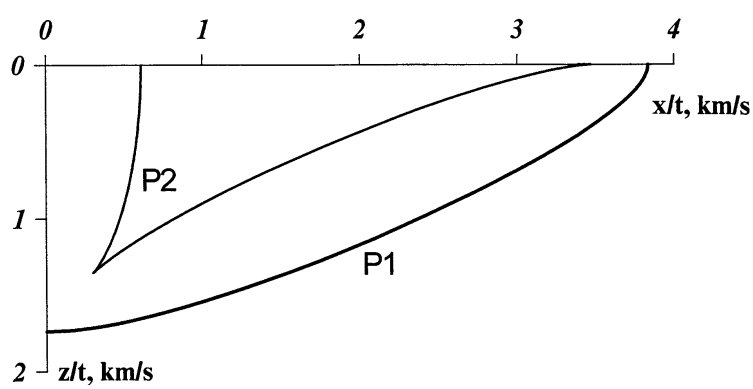


**FIG. 4. Wave fronts for Model 2 with conditions of equation (10) strictly satisfied, corresponding to complete alignment of fractures (fluid layers) parallel with the $x$-axis. The bold line corresponds to the fast quasi-longitudinal wave $P1$, the semi-bold line represents the second or "slow" quasi-longitudinal wave $P2$, and the thin line is for the quasi-shear $SV$ wave.**

Thus, the effective media model for alternating layers of fluids and solids is merely a special case of the transversely isotropic Biot model, equations (2) and (3), with

$$A = \lambda_0 b^2 (1-\varepsilon)^2 + 2\mu_2 b (1-\varepsilon)\,, \; F = (1-\varepsilon)^2 \lambda_0 b\,,$$

$$C = (1-\varepsilon)^2 \lambda_0\,, \; Q = \varepsilon (1-\varepsilon) \lambda_0\,, \; M = \varepsilon (1-\varepsilon) \lambda_0 b\,,$$

$$L = 0\,, \; N = (1-\varepsilon)\mu_2\,, \; R = \varepsilon^2 \lambda_0\,, \; P = A + 2N\,,$$

$$\alpha_1 = 1\,, \; \alpha_2 = \infty\,, \tag{15}$$

where densities $\rho_s$, $\rho_f$ and porosity $\varepsilon$ are the same parameters for both models.

The two-phase effective media model of Molotkov (1979, 1991) for layered solid-fluid systems was successfully verified by the laboratory experiments of Plona et al., (1987) which showed agreement between measured and predicted effective model velocities. The model also explains the existence of a slow interference wave, as was observed by Goloshubin et al., (1993) in thin oil-saturated layers. This wave is in fact absent in nonporous elastic layers both in theory and experimentally. Therefore we assume this model to be a good representation for interwell continuity logging. In the case of infinitely thin, fluid-filled fractures, this model is a special case of a transversely isotropic elastic medium with one of the shear moduli ($c_{44}$) equal to zero (Molotkov and Khilo, 1984).

## STRATIFIED FLUID-SOLID MEDIA WITH ANISOTROPIC ELASTIC LAYERS

The relationships derived in the previous section compare the Biot model with the effective media model for stratified fluid-solid media. These equations also apply for elastic layers possessing more complex anisotropy; the equilibrium equations of the Biot model in equation (3) remain the same. Hooke's law is represented in matrix form as

$$\left(\sigma_{xx}^{(2)}, \sigma_{yy}^{(2)}, \sigma_{zz}^{(2)}, \sigma_{yz}^{(2)}, \sigma_{xz}^{(2)}, \sigma_{xy}^{(2)}, \sigma^{(1)}\right) =$$
$$= \mathbf{C}\left(\frac{\partial u_x^{(2)}}{\partial x}, \frac{\partial u_y^{(2)}}{\partial y}, \frac{\partial u_z^{(2)}}{\partial z}, \frac{\partial u_y^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial y}, \frac{\partial u_x^{(2)}}{\partial z} + \frac{\partial u_z^{(2)}}{\partial x}, \frac{\partial u_x^{(2)}}{\partial y} + \frac{\partial u_y^{(2)}}{\partial x}, \theta\right)\,, \tag{16}$$

where **C** is a symmetric positive definite matrix with 28 independent coefficients. For the special case when the elements of **C** are given by

$$
\begin{gathered}
c_{11}=(1-\varepsilon)\left[\frac{A_{1345}^{1345}}{\chi}+(1-\varepsilon)\lambda_0\left(\frac{A_{345}^{145}}{\chi}\right)^2\right],\\
c_{12}=(1-\varepsilon)\left[\frac{A_{2345}^{1345}}{\chi}+(1-\varepsilon)\lambda_0\frac{A_{345}^{145}A_{345}^{245}}{\chi^2}\right], c_{13}=(1-\varepsilon)^2\lambda_0\frac{A_{345}^{145}}{\chi}, c_{14}=c_{15}=0,\\
c_{16}=(1-\varepsilon)\left[-\frac{A_{3456}^{1345}}{\chi}+(1-\varepsilon)\lambda_0\frac{A_{345}^{145}A_{345}^{456}}{\chi^2}\right],\qquad c_{17}=\frac{\varepsilon}{1-\varepsilon}c_{13},\\
c_{22}=(1-\varepsilon)\left[\frac{A_{2345}^{2345}}{\chi}+(1-\varepsilon)\lambda_0\left(\frac{A_{345}^{245}}{\chi}\right)^2\right], c_{23}=(1-\varepsilon)^2\lambda_0\frac{A_{345}^{245}}{\chi}, c_{24}=c_{25}=0,\\
c_{26}=(1-\varepsilon)\left[-\frac{A_{3456}^{2345}}{\chi}+(1-\varepsilon)\lambda_0\frac{A_{345}^{245}A_{345}^{456}}{\chi^2}\right],\qquad c_{27}=\frac{\varepsilon}{1-\varepsilon}c_{23},\\
c_{33}=(1-\varepsilon)^2\lambda_0, c_{36}=(1-\varepsilon)^2\lambda_0\frac{A_{345}^{456}}{\chi}, c_{37}=\frac{\varepsilon}{1-\varepsilon}c_{33},\\
c_{34}=c_{35}=c_{44}=c_{45}=c_{46}=c_{47}=c_{55}=c_{56}=c_{57}=0,\\
c_{66}=(1-\varepsilon)\left[\frac{A_{3456}^{3456}}{\chi}+(1-\varepsilon)\lambda_0\left(\frac{A_{345}^{456}}{\chi}\right)^2\right], c_{67}=\frac{\varepsilon}{1-\varepsilon}c_{36}, c_{77}=\frac{\varepsilon^2}{(1-\varepsilon)^2}c_{33},
\end{gathered}
\tag{17}
$$

with

$$
\chi=A_{345}^{345}, \lambda_0=\left(\frac{\varepsilon}{\lambda_1}+(1-\varepsilon)\frac{A_{45}^{45}}{\chi}\right)^{-1},
$$

equation (16) (transforms into the Hooke's law, representing effective media models for stratified fluid-solid media with elastic layers of arbitrary anisotropy (Molotkov and Khilo, 1983; Molotkov, 1994a). The $A^{ij}_{mn}$, $A^{ijk}_{lmn}$, $A^{ijkl}_{mnps}$ are the minors of matrix **A** for Hooke's law for material of arbitrary anisotropic elastic layers

$$
(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{yz},\sigma_{xz},\sigma_{xy})=
\mathbf{A}\left(\frac{\partial u_x}{\partial x},\frac{\partial u_y}{\partial y},\frac{\partial u_z}{\partial z},\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y},\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x},\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)
\tag{18}
$$

Similar relationships exist when the elastic layers of the stratified media have monoclinic, orthorhombic, tetragonal and hexagonal symmetry, and are presented below.

In the case of the monoclinic Biot model, matrix **C** of Hooke's law has 18 independent elements. If these elements are given by formulas

$$
\begin{gathered}
c_{11} = (1-\varepsilon)\left[a_{11} - \frac{a_{13}^2}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{13}^2}{a_{33}^2}\right] , \\
c_{12} = (1-\varepsilon)\left[a_{12} - \frac{a_{13}a_{23}}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{13}a_{23}}{a_{33}^2}\right] , \quad c_{13} = (1-\varepsilon)^2\lambda_0\frac{a_{13}}{a_{33}} , \\
c_{16} = (1-\varepsilon)\left[a_{16} - \frac{a_{13}a_{36}}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{13}a_{36}}{a_{33}^2}\right] , \quad c_{17} = \varepsilon(1-\varepsilon)\lambda_0\frac{a_{13}}{a_{33}} , \\
c_{22} = (1-\varepsilon)\left[a_{22} - \frac{a_{23}^2}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{23}^2}{a_{33}^2}\right] , \quad c_{23} = (1-\varepsilon)^2\lambda_0\frac{a_{23}}{a_{33}} , \\
c_{26} = (1-\varepsilon)\left[a_{26} - \frac{a_{23}a_{36}}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{23}a_{36}}{a_{33}^2}\right] , \quad c_{27} = \varepsilon(1-\varepsilon)\lambda_0\frac{a_{23}}{a_{33}} , \\
c_{33} = (1-\varepsilon)^2\lambda_0 , \quad c_{36} = (1-\varepsilon)^2\lambda_0\frac{a_{36}}{a_{33}} , \quad c_{37} = \varepsilon(1-\varepsilon)\lambda_0 , \\
c_{44} = c_{45} = c_{55} = 0 , \quad c_{66} = (1-\varepsilon)\left[a_{66} - \frac{a_{36}^2}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{36}^2}{a_{33}^2}\right] , \\
c_{67} = \varepsilon(1-\varepsilon)\lambda_0\frac{a_{36}}{a_{33}} , \quad c_{77} = \varepsilon^2\lambda_0 ,
\end{gathered} \tag{19}
$$

then equation (16) transforms into the Hooke's law for the effective model of stratified fluid-solid medium with monoclinic elastic layers. In equation (19), the $a_{ij}$ are the elements of matrix **A** representing Hooke's law for material with elastic layers. For monoclinic and higher symmetry $\lambda_0$ is given by

$$
\lambda_0 = \left(\frac{\varepsilon}{\lambda_1} + \frac{1-\varepsilon}{a_{33}}\right)^{-1} \tag{20}
$$

In the case of orthorhombic symmetry, the Biot model is defined by 13 independent stiffness moduli: $c_{11}$, $c_{12}$, $c_{13}$, $c_{17}$, $c_{22}$, $c_{23}$, $c_{27}$, $c_{33}$, $c_{37}$, $c_{44}$, $c_{55}$, $c_{66}$, $c_{77}$. When these elements are expressed as

$$
\begin{gathered}
c_{11} = (1-\varepsilon)\left[a_{11} - \frac{a_{13}^2}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{13}^2}{a_{33}^2}\right] , \\
c_{12} = (1-\varepsilon)\left[a_{12} - \frac{a_{13}a_{23}}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{13}a_{23}}{a_{33}^2}\right] , \\
c_{13} = (1-\varepsilon)^2\lambda_0\frac{a_{13}}{a_{33}} , \qquad c_{17} = \varepsilon(1-\varepsilon)\lambda_0\frac{a_{13}}{a_{33}}, \\
c_{22} = (1-\varepsilon)\left[a_{22} - \frac{a_{23}^2}{a_{33}} + (1-\varepsilon)\lambda_0\frac{a_{23}^2}{a_{33}^2}\right] , \\
c_{23} = (1-\varepsilon)^2\lambda_0\frac{a_{23}}{a_{33}} , \quad c_{27} = \varepsilon(1-\varepsilon)\lambda_0\frac{a_{23}}{a_{33}}, \\
c_{33} = (1-\varepsilon)^2\lambda_0 , \quad c_{37} = \varepsilon(1-\varepsilon)\lambda_0 , \quad c_{44} = c_{55} = 0 , \quad c_{66} = (1-\varepsilon)a_{66} , \quad c_{77} = \varepsilon^2\lambda_0 ,
\end{gathered} \tag{21}
$$

then Hooke's law for an orthorhombic Biot model transforms to that of an effective model of a stratified fluid-solid medium with orthorhombic elastic layers, where the $a_{ik}$ are the elastic moduli of the layers.

If the Biot model is tetragonal, then there are nine elastic constants. If these constants satisfy the conditions

$$
\begin{aligned}
c_{11} &= (1-\varepsilon)\left[a_{11} - \frac{a_{13}^2}{a_{33}} + (1-\varepsilon)\lambda_0 \frac{a_{13}^2}{a_{33}^2}\right], \\
c_{12} &= (1-\varepsilon)\left[a_{12} - \frac{a_{13}a_{23}}{a_{33}} + (1-\varepsilon)\lambda_0 \frac{a_{13}a_{23}}{a_{33}^2}\right], \\
c_{13} &= (1-\varepsilon)^2\lambda_0 \frac{a_{13}}{a_{33}}, \qquad c_{17} = \varepsilon(1-\varepsilon)\lambda_0 \frac{a_{13}}{a_{33}}, \\
c_{33} &= (1-\varepsilon)^2\lambda_0\ , \quad c_{37} = \varepsilon(1-\varepsilon)\lambda_0\ , \quad c_{77} = \varepsilon^2\lambda_0\ , \\
c_{44} &= 0\ , \quad c_{66} = (1-\varepsilon)a_{66}\ ,
\end{aligned}
\tag{22}
$$

then Hooke's law for a tetragonal Biot model transforms to Hooke's law for an effective model of stratified fluid-solid medium with tetragonal elastic layers.

The effective media model of a stratified fluid-solid medium with elastic layers possessing cubic anisotropy is a special case of the tetragonal Biot model. Whereas, when elastic layers possess hexagonal anisotropy, the effective media model is a special case of the transversely isotropic Biot model with the elastic parameters are given as

$$
\begin{aligned}
P &= (1-\varepsilon)\left[a_{11} - \frac{a_{13}^2}{a_{33}} + (1-\varepsilon)\lambda_0 \frac{a_{13}^2}{a_{33}^2}\right]\ , \\
A &= (1-\varepsilon)\left[a_{12} - \frac{a_{13}^2}{a_{33}} + (1-\varepsilon)\lambda_0 \frac{a_{13}^2}{a_{33}^2}\right]\ , \\
F &= (1-\varepsilon)^2\lambda_0 \frac{a_{13}}{a_{33}}\ , \quad M = \varepsilon(1-\varepsilon)\lambda_0 \frac{a_{13}}{a_{33}}\ , \\
C &= (1-\varepsilon)^2\lambda_0\ , \quad Q = \varepsilon(1-\varepsilon)\lambda_0\ , \quad R = \varepsilon^2\lambda_0\ , \quad L = 0\ , \\
N &= (1-\varepsilon)a_{66} = \frac{(1-\varepsilon)(a_{11}-a_{12})}{2}\ .
\end{aligned}
\tag{23}
$$

It can be seen that although there are eight Biot moduli, they are expressed by only six independent parameters (the five elastic moduli representative of the hexagonal solid, and one for the fluid).

## NEW FEATURES OF SEISMIC WAVE PROPAGATION IN ANISOTROPIC POROUS MEDIA

The investigation of models intermediate between the Biot formulation and those using effective media theory has shed light on some new features of wave propagation in anisotropic porous media:

- Wave fronts in the anisotropic Biot models display a five-valued character – up to five arrivals along chosen directions (see the shear wave front on Figure 1), and with more complex shapes than generally encountered. In this paper such features have been referred to as double loops, or loops of second order. This phenomenon does not exist in nonporous elastic anisotropic media. These loops are most obvious on the *SV* wave fronts in Figure 1. In the Biot model, such five-valued behavior occurs in nonporous media rather than the usual occurrence of triplications in anelastic media, because the order of the Christoffel equation is increased by two, giving rise to two more cusps in a single quadrant.

- There can be a longitudinal mode to shear mode transition, or vice-versa, on the same wave front in porous medium. Figure 5 shows wave fronts in the transversely isotropic Biot model. Polarizations of the two inner fronts change from pure longitudinal along one axis to pure shear along the other axis. This situation is extremely rare in elastic nonporous media and occurs only with the first longitudinal mode transitioning to the shear mode (Ledbetter and Kriz, 1982; Musgrave, 1970). In porous media this transition frequently occurs because two longitudinal waves exist which enables the transition of the second longitudinal mode to a shear mode (Figure 5). This is possible because in porous rocks the velocities of the second longitudinal wave are usually close to that of shear waves. Numerical calculations have shown that such a transition takes place providing that the following inequalities hold (Bakulin and Molotkov, 1998)

$$V_x^{P2} < V_x^{SV} \text{ and } V_z^{P2} > V_z^{SV} \text{ or } V_x^{P2} > V_x^{SV} \text{ and } V_z^{P2} < V_z^{SV}. \quad (24)$$

These inequalities are similar to those for elastic nonporous media (Helbig and Schoenberg, 1987) where the velocities of the second longitudinal waves $V_x^{P2}$ and $V_z^{P2}$ would be replaced by the velocities of the standard fast longitudinal wave. Also note that for transversely isotropic, poroelastic media, velocities $V_x^{SV}$ and $V_z^{SV}$ of the *SV*-waves parallel to the coordinate axes are different than those of transversely isotropic, elastic media. This is due to the different tortuosities along the *x* and *z* axes (Schmitt, 1989).

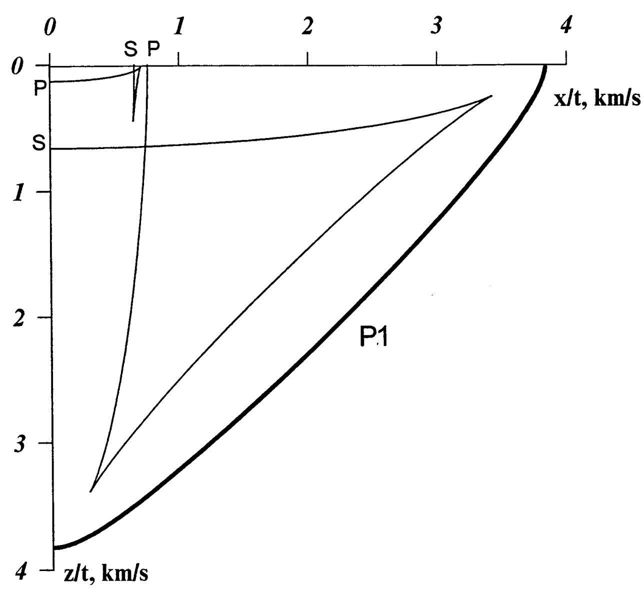


**FIG. 5. Wave fronts in the transversely isotropic Biot model (model 3) with $\alpha_1 = 3, \alpha_2 = 100$, and all other parameters as in Fig. 1. Note the second longitudinal mode to shear mode transition and vice versa on two inner fronts, where *P* represents longitudinal polarization and *S* represents shear polarization. The bold line corresponds to the fast quasi-longitudinal wave. Two inner fronts are of mixed nature.**

## DISCUSSION & CONCLUSIONS

Parameters are presented which transform the Biot (1962) model into an effective media model, showing that the effective media model of alternating fluid-solid layers is merely a special case of the Biot model. This is significant in that it provides the insight that the inner triangular wave front in the effective media model (Figure 4) corresponds to the second longitudinal Biot "slow" wave. This wave exists in all directions except directly perpendicular to the layers where it has no velocity. Therefore, along the $x$-axis the effective media model exhibits wave propagation typical of two-phase porous media, while along the $z$-axis this model exhibits monophase wave propagation typical of elastic (or fluid) media. This is because the pore space is represented by planar fractures (or fluid layers) connected along the $x$-axis and not interconnected in the $z$-axis direction. In the strict sense, a stratified fluid-solid medium can not be described directly with the Biot model since pores (the fluid layers) are separated by solid layers and therefore not connected. Nevertheless, we can say that the effective media model is one of transition between the two-phase and the monophase model, whereas in the Biot model the two sets of stresses and displacements for both phases are completely different, while in the effective media model they are partially coincident (equations (11) and (13)).

The simple model of alternating fluid and solid layers serves as an example of an anisotropic Biot medium with analytically defined elastic and geometric parameters. The relationships between models is to some extent a justification for the Biot model, constructed on the basis of physical hypotheses, whereas the model of a fluid-solid medium corresponds to the asymptotics of the wave field. Since the effective media model is a special case of the Biot model, the physical meaning of the stresses and displacements in Hooke's law are now better understood for the Biot model. For example, the stresses in Hooke's law from equation (2) are averaged over total bulk volume, whereas displacements are averaged only over the separate phase volumes (otherwise Hooke's law will not be symmetric).

Features of wave propagation in the anisotropic Biot model, such as double loops and longitudinal to shear mode transition, are seen to be unique characteristics of porous media; these features do not occur in nonporous media. These features can, therefore, be used as diagnostic tools in the detection of anisotropic porous reservoirs.

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[3] In Russian it is called Zapiski Nauchnych Seminarov LOMI (POMI since 1992).

[4] Further we use J. Sov. Math. for Journal of Soviet Mathematics which contains cover-to-cover english translation from russian of Proceedings of Saint Petersburg Branch of Steklov Mathematical Institute. Since 1993 it is called Journal of Mathematical Sciences (J. Math. Sc.).

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