

SEISMIC APPLICATIONS SERIES

VOLUME 2

**AMPLITUDE - PRESERVING
SEISMIC REFLECTION
IMAGING**

**Edited by
P. HUBRAL**

GEOPHYSICAL PRESS

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INTERNAL POINT SOURCES IN A POROELASTIC BIOT MEDIUM

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(Received March 10, 1997; revised version accepted July 2, 1997)

ABSTRACT

Molotkov, L.A. and Bakulin, A.V., 1998. Internal point sources in a poroelastic Biot medium. *In*: Hubral, P. (Ed.), *Amplitude-Preserving Seismic Reflection Imaging*, Geophysical Press Ltd., London.

For the study of wave propagation in porous media, we develop an approach for defining seismic sources and construct wave fields from ten different internal point sources in a homogeneous isotropic Biot model. All sources are defined by means of special boundary conditions with discontinuities of stresses and/or displacements on fictitious interfaces (planes or spheres). The wave fields for these sources are represented in two different forms: by relations similar to the Stokes formulae and by means of Fourier-Bessel and Laplace transforms. Derived integral identities give simple transitions from the one form to another more suitable for the concrete problem. Correctness of the obtained formulae is checked by comparing with corresponding known relations for elastic solid and fluid media which are partial cases of the Biot model. Numerically calculated radiation patterns for the far field demonstrate the peculiarities of wave propagation in porous media mainly induced by the presence of an additional slow longitudinal wave.

The paper consists of two parts. The first part is devoted to the derivation of wave fields of two elementary sources and four sources containing various force couples. In the second part of this paper, the wave fields of three point sources in a homogeneous isotropic unbounded porous Biot medium are constructed. These sources are concentrated discontinuities of displacements in both phases and are given by boundary conditions on fictitious plane interfaces. The mentioned sources can be considered as the superposition of some equivalent body forces; in addition, the resulting force and moment are equal to zero for all of them.

The far wave fields are defined by radiation patterns which determine the displacements in both phases on all three wave fronts. These radiation patterns are computed and analyzed in the case of four sources for some parameters.

KEYWORDS: poroelastic, Biot, porous media, elementary sources, point sources, force couples, radiation patterns, discontinuity.

PART I: ELEMENTARY SOURCES AND FORCE COUPLES

INTRODUCTION

Biot theory has a growing applications in solving different geophysical problems. Most promising results are expected from using it in acoustic logging for determination of various properties of porous formations (Schmitt, 1989). In order to achieve a better understanding of wave propagation, in many cases it is necessary to know the wave fields and radiation patterns of the seismic sources used. As a starting point, sources in a homogeneous isotropic porous medium should be considered.

Burridge and Vargas (1979) were the first who considered a point force applied to the skeleton of a porous medium. Norris (1985) defined a point force in the fluid phase which leads to contradictions, as was shown by Bonnet (1987). The latter author concluded that in the fluid phase one can define only the centre of compression. In general, we may say that only irrotational (potential) forces may be posed in the fluid phase (Bakulin, 1996) in the same way as is done in a uniform fluid. So the fluid phase has one degree of freedom and only one scalar potential is needed to define any source acting in the fluid phase (Dai et al., 1995). A formal thermoelastic analogy was used by Bonnet (1987) and Boutin et al. (1987) in order to obtain the Green's function for an isotropic unbounded Biot model.

In all references, with the exception of that by Bakulin (1996), the sources were given on the right-hand side of wave equations written for a Biot medium. In contrast, in this paper we use a more natural approach (Molotkov and Bakulin, 1995) which has clear relations with conventional elasticity and does not demand any reformulation of the Biot equations as in Bonnet (1987) and Boutin et al. (1987). In this approach we define all sources as discontinuities of stresses and displacements on fictitious interfaces. Useful relations between two ways of defining sources in a Biot model were given by Bakulin and Molotkov (1996) and Bakulin (1997).

By means of our approach we consider the following nine cases of internal seismic point sources in the poroelastic Biot model:

1. The point force acting on the solid phase (the first elementary source);
2. The centre of pressure applied to the fluid phase (second elementary source);
3. The momentless force couple;
4. Three force couples acting along the three coordinate axes;
5. Force couple with a moment;
6. Two force couples with a moment (centre of rotation);
7. The source as the longitudinal discontinuity of displacement in the solid phase;

8. The source as the longitudinal discontinuity of displacement in the fluid phase;
9. The source as the transverse discontinuity of displacement in the solid phase.

Wave fields are derived for every source. The sources 1 and 2 are elementary. The wave fields of all others could be derived on the basis of fields of the first and second sources. The fields of the sources 3-6 are established by means of the first source, although the fields of sources 4 and 6 may also be obtained in other ways. The fields of sources 7, 8 and 9 are derived independently.

The wave fields are represented in two forms: by relations similar to Stokes formulae and by means of Fourier-Bessel and Laplace transforms. The first representation is of interest for the generalization of the ray method on the Biot medium and permits construction of the radiation patterns of the displacements in the far zone. The second representation allows the process of reflection and refraction on the interfaces of layered porous media to be considered. By means of simple operations the Fourier-Bessel and Mellin integrals used can be written in the inverse order or replaced by other integrals. To establish the connection between both representations, a set of integral identities is derived in this paper which provides immediate transformation from one form to another.

In certain conditions, the waves in the solid and fluid phases may propagate independently and Biot equations are decomposed in two sets: one for fluid and another for solid. In this case, all expressions for wave fields are coincident with corresponding known expressions for fluid and solid media.

THE MAIN EQUATIONS OF A HOMOGENEOUS ISOTROPIC BIOT MODEL

Let us consider an unbounded homogeneous isotropic two-phase Biot medium, in which the superscripts 1 and 2 are used for fluid and elastic phases, respectively. Displacements $u_i^{(1)}$, $u_i^{(2)}$ and stresses $\sigma_{ij}^{(2)}$ satisfy equations of Hooke's law

$$\sigma_{xx}^{(2)} = P(\partial u_x^{(2)}/\partial x) + A(\partial u_y^{(2)}/\partial y) + A(\partial u_z^{(2)}/\partial z) + M\theta,$$

$$\sigma_{yy}^{(2)} = A(\partial u_x^{(2)}/\partial x) + P(\partial u_y^{(2)}/\partial y) + A(\partial u_z^{(2)}/\partial z) + M\theta,$$

$$\sigma_{zz}^{(2)} = A(\partial u_x^{(2)}/\partial x) + A(\partial u_y^{(2)}/\partial y) + P(\partial u_z^{(2)}/\partial z) + M\theta,$$

$$\sigma^{(1)} = M(\partial u_x^{(2)}/\partial x) + M(\partial u_y^{(2)}/\partial y) + M(\partial u_z^{(2)}/\partial z) + \bar{R}\theta, \quad (1)$$

$$\sigma_{yz}^{(2)} = L[(\partial u_y^{(2)}/\partial z) + (\partial u_z^{(2)}/\partial y)], \quad \sigma_{xz}^{(2)} = L[(\partial u_x^{(2)}/\partial z) + (\partial u_z^{(2)}/\partial x)],$$

$$\sigma_{xy}^{(2)} = L[(\partial u_x^{(2)}/\partial y) + (\partial u_y^{(2)}/\partial x)], \quad \theta = (\partial u_x^{(1)}/\partial x) + (\partial u_y^{(1)}/\partial y) + (\partial u_z^{(1)}/\partial z),$$

with $P = A + 2L$ and equations of a continuous medium

$$\begin{aligned} (\partial \sigma_{xx}^{(2)}/\partial x) + (\partial \sigma_{xy}^{(2)}/\partial y) + (\partial \sigma_{xz}^{(2)}/\partial z) &= \rho'_{11} \ddot{u}_x^{(2)} + \rho'_{12} \ddot{u}_x^{(1)}, \\ (\partial \sigma_{xy}^{(2)}/\partial x) + (\partial \sigma_{yy}^{(2)}/\partial y) + (\partial \sigma_{yz}^{(2)}/\partial z) &= \rho'_{11} \ddot{u}_y^{(2)} + \rho'_{12} \ddot{u}_y^{(1)}, \\ (\partial \sigma_{xz}^{(2)}/\partial x) + (\partial \sigma_{xy}^{(2)}/\partial y) + (\partial \sigma_{zz}^{(2)}/\partial z) &= \rho''_{11} \ddot{u}_z^{(2)} + \rho''_{12} \ddot{u}_z^{(1)}, \\ (\partial \sigma^{(1)}/\partial x) &= \rho'_{12} \ddot{u}_x^{(2)} + \rho'_{22} \ddot{u}_x^{(1)}, \\ (\partial \sigma^{(1)}/\partial y) &= \rho'_{12} \ddot{u}_y^{(2)} + \rho'_{22} \ddot{u}_y^{(1)}, \\ (\partial \sigma^{(1)}/\partial z) &= \rho''_{12} \ddot{u}_z^{(2)} + \rho''_{22} \ddot{u}_z^{(1)}. \end{aligned} \quad (2)$$

Parameters ρ_{11} , ρ_{12} and ρ_{22} are determined by the formulae

$$\rho_{11} = \rho_s(1 - \epsilon) + \epsilon \rho_f(\alpha - 1), \quad \rho_{12} = -\rho_f \epsilon(\alpha - 1) \text{ and } \rho_{22} = \rho_f \epsilon \alpha, \quad (3)$$

where ρ_s and ρ_f are the densities of the solid and fluid phases, ϵ is the porosity and α the tortuosity of the pore space.

In the given medium there is a connection between waves in both phases and only in the case

$$\rho_{12} = 0, \quad M = 0 \quad (4)$$

do the waves in the elastic and fluid phases propagate independently and they are separately described by the equations of the theories of elasticity and hydroacoustics, respectively.

To construct the wave fields, the scalar potentials φ_1 , φ_2 and the vector potential ψ are used. Displacement vectors are expressed by the relations

$$u^{(1)} = \text{grad} \varphi_1 - (\rho_{12}/\rho_{22}) \text{rot} \psi, \quad u^{(2)} = \text{grad} \varphi_2 + \text{rot} \psi \quad (5)$$

in terms of potentials φ_1 , φ_2 and ψ , which satisfy the equations

$$\begin{aligned} P \Delta \varphi_2 + M \Delta \varphi_1 &= \rho_{11}(\partial^2 \varphi_2 / \partial t^2) + \rho_{12}(\partial^2 \varphi_1 / \partial t^2), \\ M \Delta \varphi_2 + \bar{R} \Delta \varphi_1 &= \rho_{12}(\partial^2 \varphi_2 / \partial t^2) + \rho_{22}(\partial^2 \varphi_1 / \partial t^2), \\ \Delta \psi &= (\delta / \rho_{22} L)(\partial^2 \psi / \partial t^2), \end{aligned} \quad (6)$$

where $\delta = \rho_{11}\rho_{22} - \rho_{12}^2$. For axially symmetrical sources, the vector potential

ψ is expressed by the relations

$$\psi = \psi_\theta \theta_1 \quad (7)$$

in the cylindrical system of coordinates r, z, θ . Taking into account formula (7), the displacements and stresses are represented by the formulae

$$\begin{aligned} u_z^{(1)} &= (\partial \varphi_1 / \partial z) - (\rho_{12}/\rho_{22})[(\partial \psi_\theta / \partial r) + (\psi_\theta / r)], \\ u_r^{(1)} &= (\partial \varphi_1 / \partial r) + (\rho_{12}/\rho_{22})(\partial \psi_\theta / \partial z), \\ u_z^{(2)} &= (\partial \varphi_2 / \partial z) + (\partial \psi_\theta / \partial r) + (\psi_\theta / r), \\ u_r^{(2)} &= (\partial \varphi_2 / \partial r) - (\partial \psi_\theta / \partial z), \\ \sigma^{(1)} &= \rho_{12}(\partial^2 \varphi_2 / \partial t^2) + \rho_{22}(\partial^2 \varphi_1 / \partial t^2), \\ \sigma_{zz}^{(2)} &= \rho_{11}(\partial^2 \varphi_2 / \partial t^2) + \rho_{12}(\partial^2 \varphi_1 / \partial t^2) \\ &\quad + 2L[(\partial^2 \varphi_2 / \partial r^2) + (1/r)(\partial \varphi_2 / \partial r) - (\partial^2 \psi_\theta / \partial r \partial z) - (1/r)(\partial \psi_\theta / \partial z)], \\ \sigma_{rz}^{(2)} &= L[2(\partial^2 \varphi_1 / \partial r \partial z) - (\partial^2 \psi_\theta / \partial z^2) + (\partial^2 \psi_\theta / \partial r^2) + (1/r)(\partial \psi_\theta / \partial r) - \psi_\theta / r^2]. \end{aligned} \quad (8)$$

1. THE POINT FORCE ACTING ON THE SOLID PHASE (THE FIRST ELEMENTARY SOURCE)

The first of the elementary sources is defined by the relations

$$[\sigma_{zz}^{(2)}] = [\delta(r)f(t)]/2\pi r, \quad [\sigma^{(1)}] = [\sigma_{rz}^{(2)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0, \quad (9)$$

where $\delta(r)$ is the Dirac function. The symbol $[]$ denotes the jump of a function in the square brackets at $z = 0$. The right-hand side of the first formula of (9) expresses the point force applied to the solid phase. Function $f(t)$ that defines the dependence of intensity from time for all the sources satisfies a condition

$$f(t) = 0 \quad \text{as } t < 0. \quad (10)$$

The solutions of eqs. (6) are represented by the formulae

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} &= \text{sign} z \int_0^\infty \frac{J_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} X_1 e^{-k|z|\alpha_1} + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} X_2 e^{-k|z|\alpha_2} \right] e^{k\tau\eta} d\eta, \\ \psi_\theta &= \int_0^\infty \frac{J_1(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Y e^{-k|z|\beta} e^{k\tau\eta} d\eta. \end{aligned} \quad (11)$$

This representation of the wave fields is widely used in elastic and fluid media and its foundation may be found in Petrashen (1959) or Molotkov (1984). On the basis of boundary conditions (9), functions X_1 , X_2 , and Y are expressed by the relations

$$X_1 = -\gamma_{22}F(k\eta)/4\pi\eta^2\delta A_{21}, \quad X_2 = \gamma_{21}F(k\eta)/4\pi\eta^2\delta A_{21},$$

$$Y = \rho_{22}F(k\eta)/4\pi\eta^2\beta\delta, \quad F(k\eta) = \int_0^\infty f(t)e^{-k\eta t}dt, \quad (12)$$

where

$$A_{1,2} = (\rho_{12}v_{1,2}^2 - M)/(P - \rho_{11}v_{1,2}^2) = (\rho_{22}v_{1,2}^2 - \bar{R})/(M - \rho_{12}v_{1,2}^2),$$

$$\delta = \rho_{11}\rho_{22} - \rho_{12}^2, \quad \gamma_{21} = \rho_{12}A_1 + \rho_{22}, \quad \gamma_{22} = \rho_{12}A_2 + \rho_{22},$$

$$A_{21} = A_2 - A_1, \quad \alpha_1 = \sqrt{1 + (\eta^2/v_1^2)}, \quad \alpha_2 = \sqrt{1 + (\eta^2/v_2^2)},$$

$$\beta = \sqrt{1 + (\eta^2/v_3^2)}, \quad (13)$$

and v_1 , v_2 and v_3 are the velocities of two longitudinal waves and one transverse wave

$$v_{1,2}^2 = [P\rho_{22} + \bar{R}\rho_{11} - 2M\rho_{12} \pm \sqrt{(P\rho_{22} + \bar{R}\rho_{11} - 2M\rho_{12})^2 - 4\delta(P\bar{R} - M^2)}]/2\delta,$$

$$v_3 = \sqrt{L\rho_{22}/\delta}. \quad (14)$$

The expressions (11) - (13) were obtained by Molotkov and Bakulin (1995).

For deriving the Stokes formulae, the known identity

$$e^{-\gamma R/R} = \int_0^\infty kJ_0(kr)[e^{-|z|\sqrt{(k^2+\gamma^2)}}/\sqrt{(k^2+\gamma^2)}]dk, \quad (15)$$

with spherical radius $R = \sqrt{(r^2 + z^2)}$, is used. On the basis of relation (15), we find the new formula

$$f(t - R/v)/R = \int_0^\infty kJ_0(kr)dk/2\pi i \cdot \int_{\sigma-i\infty}^{\sigma+i\infty} [F(k\eta)e^{k(\eta-|z|/\alpha)}]d\eta. \quad (16)$$

If we differentiate Eqs. (16) over r , z , x , y and t , we obtain additional formulae which allow us to establish connections between the Stokes formulae and the integral relations in the case of every source. As a result, we get the formulae

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{4\pi\delta A_{21}} \left(\frac{\partial}{\partial z} \frac{1}{R} \right) \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \gamma_{22}g_1 - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \gamma_{21}g_2 \right],$$

$$\psi_\theta = -\frac{\rho_{22}}{4\pi\delta} \left(\frac{\partial}{\partial r} \frac{1}{R} \right) g_3, \quad g_i = \int_{R/v_i}^t \tau f(t-\tau)d\tau \quad (i=1,2,3) \quad (17)$$

for the potentials φ_1 , φ_2 and ψ .

2. THE CENTRE OF PRESSURE APPLIED TO THE FLUID PHASE (THE SECOND ELEMENTARY SOURCE)

This source is given on a sphere $R = R_0$ by the relation

$$[\sigma^{(1)}] = 3f(t)/4\pi R_0^3, \quad [\sigma_{RR}^{(2)}] = 0, \quad [u_R^{(1)}] = [u_R^{(2)}] = 0, \quad (18)$$

where the symbol $[]$ denotes the jump of functions in the square brackets. For such a source we define the wave field in which the transition $R_0 \rightarrow 0$ is fulfilled. The wave field after this transition is represented by the formulae:

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \int_0^\infty \frac{kJ_0(kr)dk}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} X_1 e^{-k|z|\alpha_1} + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} X_2 e^{-k|z|\alpha_2} \right] e^{k\eta t} d\eta, \quad (19)$$

where

$$X_1 = \gamma_{12}F(k\eta)/4\pi v_1^2\delta\alpha_1 A_{21}, \quad X_2 = -\gamma_{11}F(k\eta)/4\pi v_2^2\delta\alpha_2 A_{21},$$

$$\gamma_{11} = \rho_{11}A_1 + \rho_{12}, \quad \gamma_{12} = \rho_{11}A_2 + \rho_{12}, \quad (20)$$

or

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{4\pi\delta A_{21}R} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{\gamma_{12}}{v_1} f_1 - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\gamma_{11}}{v_2} f_2 \right], \quad \psi_\theta = 0. \quad (21)$$

If conditions (4) are fulfilled, then $\varphi_2 = 0$, $\psi_\theta = 0$ and φ_1 correspond to the expressions of potentials in fluid from the centre of pressure (Molotkov, 1986).

3. THE MOMENTLESS FORCE COUPLE

The Biot medium is excited by two opposite-directed forces, applied at points $r = 0$, $z = R_0$ and $r = 0$, $z = -R_0$. The two sources are defined by the conditions

$$[\sigma_{zz}^{(2)}] = \pm \delta(r)f(t)/4\pi R_0, \quad [\sigma_{zz}^{(2)}] = [\sigma^{(1)}] = 0, \quad [u_z^{(1)}] = [u_z^{(2)}] = [u_r^{(2)}] = 0 \quad (22)$$

for $z = \pm R_0$. Symbol $[]$ expresses the discontinuities of the functions in the square bracket on the planes $z = \pm R_0$. In the wave that satisfies conditions (22)

and (10), it is necessary to carry out the transition $R_0 \rightarrow 0$. The investigation of this source done by Molotkov and Bakulin (1995) shows that for the transition from the wave field of the source (9) into the wave field of source (22), it is sufficient to differentiate expressions (11), (17) over z and to change a sign in them.

4. THREE FORCE COUPLES ACTING ALONG THE THREE COORDINATE AXES

The Biot medium is excited by three force couples acting along the three coordinate axes

$$\begin{aligned} [\sigma_{zz}^{(2)}] &= \pm \delta(x)\delta(y)f(t)/2R_0, \quad [\sigma_{xz}^{(2)}] = [\sigma_{yz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_z^{(1)}] &= [u_z^{(2)}] = [u_x^{(2)}] = [u_y^{(2)}] = 0 \text{ for } z = \pm R_0; \\ [\sigma_{xx}^{(2)}] &= \pm \delta(y)\delta(z)f(t)/2R_0, \quad [\sigma_{xy}^{(2)}] = [\sigma_{xz}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_x^{(1)}] &= [u_x^{(2)}] = [u_y^{(2)}] = [u_z^{(2)}] = 0 \text{ for } x = \pm R_0; \\ [\sigma_{yy}^{(2)}] &= \pm \delta(x)\delta(z)f(t)/2R_0, \quad [\sigma_{yz}^{(2)}] = [\sigma_{xy}^{(2)}] = [\sigma^{(1)}] = 0, \\ [u_y^{(1)}] &= [u_y^{(2)}] = [u_x^{(2)}] = [u_z^{(2)}] = 0 \text{ for } y = \pm R_0. \end{aligned} \quad (23)$$

In the wave field that satisfies conditions (23), it is necessary to fulfill the transition $R_0 \rightarrow 0$. The wave field can be expressed for each force couple. After addition of these wave fields, we obtain the following formulae for potentials

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = -\frac{1}{4\pi\delta A_{21}R} \left[\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{\gamma_{22}}{v_1^2} f_1 - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\gamma_{21}}{v_1^2} f_2 \right], \quad \psi = 0. \quad (24)$$

By means of integral transforms, these potentials are given by (19) with the following X_1 and X_2

$$X_1 = \gamma_{22}F(k\eta)/4\pi(A_2 - A_1)\delta v_1^2\alpha_1, \quad X_2 = \gamma_{21}F(k\eta)/4\pi(A_2 - A_1)\delta v_2^2\alpha_2. \quad (25)$$

The same wave field arises from the centre of pressure, acting on the solid phase

$$[\sigma_{RR}^{(2)}] = 3f(t)/4\pi R_0^3, \quad [\sigma^{(1)}] = 0, \quad [\sigma_R^{(1)}] = [\sigma_R^{(2)}] = 0, \quad (26)$$

where symbol $[]$ in Eqs. (26) denotes the jump at $R = R_0$ and in the wave field satisfying conditions (26) it is necessary to fulfill the transition $R_0 \rightarrow 0$. Thus the three force couples mentioned are equivalent to the centre of pressure acting on the solid phase. Such a relation between these sources exists also in the case of elastic media.

5. FORCE COUPLE WITH A MOMENT

The Biot medium is excited by couple of forces applied at points $x = R_0$, $y = z = 0$ and $x = -R_0$, $y = z = 0$ (Fig. 1). These forces are given by the conditions

$$[\sigma_{zz}^{(2)}] = [\delta(x - R_0)\delta(y)f(t)/4\pi R_0] - [\delta(x + R_0)\delta(y)f(t)/4\pi R_0], \quad (27)$$

and in the obtained wave field it is necessary to fulfill the passage to the limit as $R_0 \rightarrow 0$. In order to write the limited wave field, it is necessary to use the expressions (17) of the wave field of one force. In these expressions we differentiate over x and change a sign. Then we obtain the following formulae for the potentials

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = -\begin{pmatrix} 1 \\ A_1 \end{pmatrix} \frac{\gamma_{22}xz}{4\pi\delta A_{21}} \left[\frac{3g_1}{R^5} + \frac{f_1}{v_1^2 R^3} \right] + \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \frac{\gamma_{21}xz}{4\pi\delta A_{21}} \left[\frac{3g_2}{R^5} + \frac{f_2}{v_1^2 R^3} \right]. \quad (28)$$

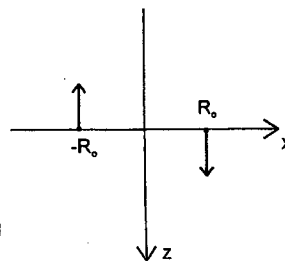


Fig. 1. Force couple with a moment in the solid phase (source N 6).

6. TWO FORCE COUPLES WITH A MOMENT (CENTRE OF ROTATION)

The Biot medium is excited by two force couples (Fig. 2), given by the conditions

$$\begin{aligned} [\sigma_{xx}^{(2)}] &= [\delta(z + R_0)\delta(y)f(t)/4\pi R_0] - [\delta(z - R_0)\delta(y)f(t)/4\pi R_0], \text{ for } z = 0 \\ [\sigma_{zz}^{(2)}] &= [\delta(x - R_0)\delta(y)f(t)/4\pi R_0] - [\delta(x + R_0)\delta(y)f(t)/4\pi R_0], \text{ for } x = 0. \end{aligned} \quad (29)$$

By methods used in consideration of the last sources we obtain, after the transition $R_0 \rightarrow 0$, the following expressions

$$\begin{aligned}
\psi_x &= -(\rho_{22}/4\pi\delta)[(3xy/R^5)g_3 + (xy/v_3^2 R^3)f_3] , \\
\psi_y &= (\rho_{22}/4\pi\delta)[\{(x^2 - 2y^2 + z^2)/R^5\}g_3 + \{(x^2 + z^2)/v_3^2 R^3\}f_3] , \\
\psi_z &= -(\rho_{22}/4\pi\delta)[(3yz/R^5)g_3 + (yz/v_3^2 R^3)f_3] . \quad \varphi_1 = \varphi_2 = 0 .
\end{aligned} \quad (30)$$

Formulae (30) can be simplified when we use a property of the vector potential: if one adds the gradient of an arbitrary function to a vector potential, then the wave field does not change. As such a function we choose:

$$\phi = -(\rho_{22}/4\pi\delta)(y/R^3) \int_{R/v_3}^t \tau f(t - \tau) d\tau . \quad (31)$$

Then the expressions (30) are replaced by the relations

$$\varphi_1 = \varphi_2 = \psi_x = \psi_z = 0 , \quad \psi_y = (1/v_3^2 R)f(t - R/v_3) , \quad (32)$$

describing the wave field caused by the centre of rotation around the y-axis.

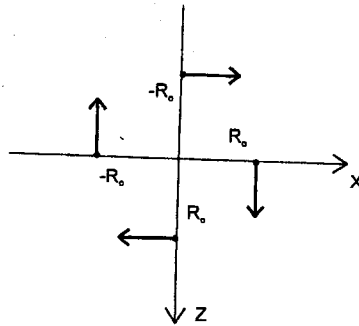


Fig. 2. Two force couples with a moment (center of rotation - source N 7).

PART II. THE SOURCES AS DISCONTINUITIES OF DISPLACEMENTS

INTRODUCTION

In the second part of this paper, we continue to define the wave fields excited by internal point sources in a homogeneous, isotropic porous Biot medium. We consider three sources as concentrated discontinuities of displacements. Such sources are of great interest for the study of wave propagation in earthquake and exploration seismology. The sources, as in the first part, are given by boundary conditions on fictitious interfaces. We continue the numbering of formulae and figures from the first part for easy referencing.

7. THE SOURCE AS THE LONGITUDINAL DISCONTINUITY OF DISPLACEMENTS IN THE SOLID PHASE

The Biot medium is excited by the longitudinal discontinuity of displacement in the solid phase. This source is given by the conditions

$$\begin{aligned}
[u_z^{(2)}] &= f(t)\delta(r)/2\pi r , \quad [\sigma_{zz}^{(2)}] = [\sigma_{rz}^{(2)}] = [\sigma_{\theta z}^{(2)}] = [\sigma^{(1)}] = 0 , \\
[u_z^{(1)}] &= [u_r^{(2)}] = [\sigma_\theta^{(2)}] = 0 ,
\end{aligned} \quad (33)$$

where $\delta(r)$ is the Dirac function, the symbol $[]$ denotes the discontinuity of the function in the square brackets at $z = 0$.

In accordance with Bakulin (1997), the wave field can be represented as a superposition of wave fields of three sources:

1. The momentless force couple acting along the z-axis (source N 3, (22) with additional multiplier $-2L$),
2. The centre of pressure acting on the elastic phase (source N 4, (23) with additional multiplier $-A$), and
3. The centre of pressure acting on the fluid phase (source N 2, (18) with additional multiplier $-M$).

8. THE SOURCE AS THE LONGITUDINAL DISCONTINUITY OF DISPLACEMENTS IN THE FLUID PHASE

The Biot medium is excited by a longitudinal discontinuity of displacement in the fluid phase. This source is given by the conditions

$$\begin{aligned}
[u_z^{(1)}] &= \delta(r)f(t)/2\pi r , \quad [u_z^{(2)}] = [u_r^{(2)}] = 0 , \\
[\sigma_{zz}^{(2)}] &= [\sigma_{rz}^{(2)}] = [\sigma^{(1)}] = 0 , \quad \text{for } z = 0 .
\end{aligned} \quad (34)$$

In accordance with Bakulin (1997), the wave field can be represented as a superposition of wave fields of two sources:

1. The centre of pressure acting on the elastic phase (source N 4, (23) with additional multiplier M),
2. The centre of pressure acting on the fluid phase (source N 2, (18) with additional multiplier $\bar{R}$).

The displacements in this general case are expressed by the formulae

$$\begin{pmatrix} u_1^{(1)} \\ u_2^{(2)} \end{pmatrix} = \frac{z}{4\pi A_{21} R^2} \left\{ \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{A_2}{R} f_1 + \frac{A_2}{v_1} f_1' \right] - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{A_1}{R} f_2 + \frac{A_1}{v_1} f_2' \right] \right\},$$

$$\begin{pmatrix} u_1^{(2)} \\ u_2^{(2)} \end{pmatrix} = \frac{x}{4\pi A_{21} R^2} \left\{ \begin{pmatrix} 1 \\ A_1 \end{pmatrix} \left[\frac{A_2}{R} f_1 + \frac{A_2}{v_1} f_1' \right] - \begin{pmatrix} 1 \\ A_2 \end{pmatrix} \left[\frac{A_1}{R} f_2 + \frac{A_1}{v_1} f_2' \right] \right\}. \quad (35)$$

9. THE SOURCE AS THE TRANSVERSE DISCONTINUITY OF DISPLACEMENTS IN THE SOLID PHASE

The Biot medium is excited by a discontinuity of $u_x^{(2)}$ in the solid phase. This source is given by the conditions

$$\begin{aligned} [u_x^{(2)}] &= \delta(r)f(t)/2\pi r, \quad [u_y^{(2)}] = 0, \quad [\sigma_{xz}^{(2)}] = [\sigma_{yz}^{(2)}] = 0, \\ [u_z^{(2)}] &= [u_x^{(1)}] = 0, \quad [\sigma_{xz}^{(2)}] = [\sigma_{yz}^{(2)}] = 0. \end{aligned} \quad (36)$$

for $z = 0$. In deriving the wave field, it is helpful to use the property that potentials, displacements and stresses are even or odd functions of z . Since $u_z^{(1)}$, $u_z^{(2)}$, $\sigma_{xz}^{(2)}$ and $\sigma_{yz}^{(2)}$ are even and continuous, then the four last conditions in (36) are carried out automatically and the problem is reduced to fulfilling only the first four conditions in (36). Therefore, we use the relations

$$\begin{aligned} u_x^{(2)} &= (\partial\varphi_2/\partial r) + (1/r)(\partial\psi_2/\partial\theta) - \cos\theta(\partial\psi_y/\partial z), \\ u_y^{(2)} &= (1/r)(\partial\varphi_2/\partial\theta) - (\partial\psi_2/\partial\theta) + \sin\theta(\partial\psi_y/\partial z), \\ \sigma^{(1)} &= \rho_{12}(\partial^2\varphi_2/\partial t^2) + \rho_{22}(\partial^2\varphi_1/\partial t^2), \\ \sigma_{zz}^{(2)} &= \rho_{11}(\partial^2\varphi_2/\partial t^2) + \rho_{12}(\partial^2\varphi_1/\partial t^2) - 2L[(\partial^2\varphi_2/\partial r^2) + (1/r)(\partial\varphi_2/\partial r) + \\ &\quad (1/r^2)(\partial^2\varphi_2/\partial\theta^2) - \cos\theta(\partial^2\psi_y/\partial r\partial z) + (\sin\theta/r)(\partial^2\psi_y/\partial\theta\partial z)], \end{aligned} \quad (37)$$

permitting us to express the displacements and stresses involved in the four first conditions (36) in terms of the potentials.

CONCLUSION

The obtained wave fields of the ten seismic point sources provide a possibility to carry out the detailed study of wave propagation in a homogeneous isotropic porous Biot medium. In particular, these expressions permit the derivation of the wave fields in the case of more complicated point sources (for

example, quadruple), to transfer to distributed sources, and to compare the wave fields excited by the different point sources. On the basis of the obtained wave fields of the considered sources and the coefficients of reflection and refraction on interfaces, the wave processes in stratified media containing porous layers can be studied. The obtained wave fields from the point sources are a basis for the use of the ray method in the Biot medium.

The application of boundary conditions for defining sources can be used also for determination of the wave field in the Biot medium with losses caused by relative motion of fluid and solid. In these cases, the expressions for wave fields proved to be much more complicated. Therefore, taking into account a dissipation is another problem which will be considered in future publications.

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APPENDIX

RADIATION PATTERNS (DIAGRAMS) OF SOURCES 1, 7, 8, AND 9

In this section the displacements are investigated at the far field where the spherical radius R is taken large and the only terms decreasing as $1/R$ with increasing R remain. In this region the wave field is composed of individual waves P_1 , P_2 and S . In addition, the first two waves are strictly longitudinal and the third wave is strictly transverse. Indeed, the directions of displacement vectors on the wave fronts of waves P_1 and P_2 coincide with the direction of the ray or are opposite to this direction. The displacement vector on the front of wave S is perpendicular to the ray. We also assume that the time function of source $f(t)$ equals zero at $t < 0$ and satisfies condition $f(0) \neq 0$ for source (9) and condition $f'(0) \neq 0$, when sources (33), (34) or (36) are applied. As the actions of sources in different directions showed to be distinct, it is useful to construct radiation patterns for every source. These patterns define graphs of dependencies of displacements on the wave fronts from the angle in a polar system of coordinates on the plane xz . In addition, the origin coincides with the position of the source.

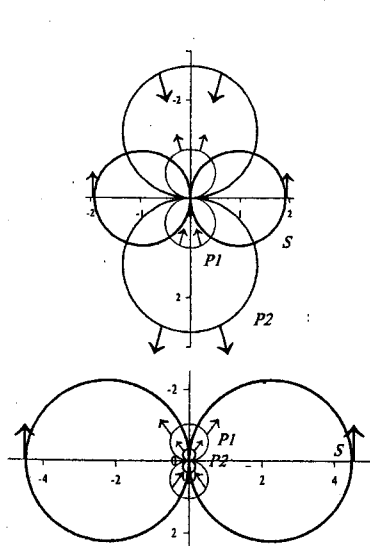


Fig. 3. Radiation patterns of source N 1 (the concentrated force acting on the solid phase).

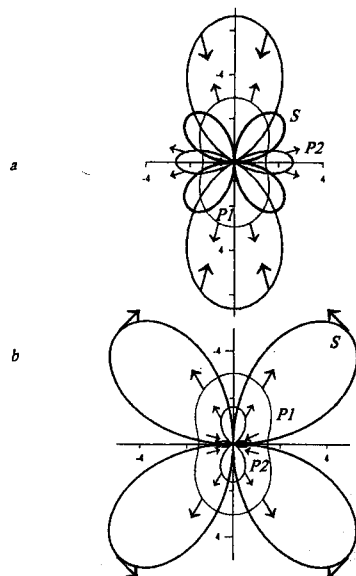


Fig. 4. Radiation patterns of source N 8 (longitudinal discontinuity of displacement in the solid phase).

Diagrams of relative displacements on fronts from the sources being a vertical force (9), the discontinuity of normal displacements in the skeleton (33), the discontinuity of normal displacement in fluid (34), and the jump of tangential displacements in the solid phase (36) are as shown in Figs. 3-6. In every figure two diagrams marked by the letters a and b and relating to fluid and solid phases are represented. The arrows on the diagrams show the displacement directions. The thin line represents the diagram of the longitudinal wave P_1 , the semi-bold line shows the diagram of the longitudinal wave P_2 , and the bold line represents the diagram of the transverse wave S . For definiteness we note that all the radiation patterns were constructed for an isotropic Biot medium with the parameters $A = 4.43$ GPa, $L = 2.77$, $Q = 0.74$, $R = 0.6$ GPa, $\alpha = 1.75$, $\epsilon = 0.26$, $\rho_s = 2650$ kg/m³, $\rho_f = 1000$ kg/m³. The propagation velocities and coefficients A_1 , A_2 and $-\rho_{22}/\rho_{12}$, determining the ratios of amplitudes of waves P_1 , P_2 and S in the skeleton and the fluid phase have the values $v_1 = 2330$ m/sec, $v_2 = 1030$ m/sec, $v_3 = 1160$ m/sec, $A_1 = 1.04$, $A_2 = -0.12$, $-\rho_{22}/\rho_{12} = +2.33$ in formulae (14), (3) and (13). The distinctive peculiarity of all diagrams is an opposite direction of displacement in both phases for the second longitudinal wave P_2 . In addition, the skeleton displacements in this wave are larger by approximately one order than in the fluid. This means that the biggest relative displacement always occurs for the propagation of the second longitudinal wave. As it is known that the relative displacement is a reason for the seismo-electric effect, we may conclude that it is primarily caused by the second longitudinal wave P_2 . We also note rather great displacements in the wave S in the fluid phase. These displacements are only 2.3 times smaller than in the skeleton. The diagram in Fig. 3b for waves P_1 and S is close to the diagrams for waves P and S from a vertical force in an elastic medium (Aki and Richards, 1980), but the directions of displacements in these cases prove to be opposite. This difference is connected with fact that in this paper the applied force is given by a jump of stress $\sigma_{zz}^{(2)}$, whereas in Aki and Richards (1980) the source is defined by a body force. The mentioned difference is in agreement with Bakulin (1997), where body forces equivalent to the jumps of displacements and stresses were established.

The jump of $u_z^{(2)}$ is characterized by the diagram given by Fig. 4. For wave P_1 the diagram is a superposition of diagrams of the pressure centre and the momentless couple of vertical forces (Bakulin, 1997). The diagram of P_1 is close to the diagram of a P -wave caused by a jump of u_z in the elastic one-phase medium. For wave P_2 , the displacements from the force couple have larger absolute values than the displacements from the pressure centres, but these displacements have the opposite direction. These facts stipulate a four-petal diagram with four directions, along which the displacements vanish. The diagrams of wave S are typical diagrams from the momentless couple of vertical forces (White, 1983). In Fig. 4b the displacements in wave P_2 were increased by factor 2 for clearness.

The jump of $u_z^{(1)}$ excites a spherically symmetrical longitudinal field (Fig. 5) in both phases. If we assume that the displacement in the P1-wave in the fluid phase equals one, then the displacement in the wave P2 in the same phase is equal to 19 units. Respectively, the displacements of the skeleton in waves P1 and P2 are equal to 1.8 and 2.75 units. Thus, source (34) is the best generator of wave P2. All displacements are directed from the centre of the source with the exception of the skeleton displacement in the wave P2.

The jump $u_x^{(2)}$, which is equivalent to the double momentless force couple in the plane xz , is characterized by the four-petal diagrams of P1, P2 and S waves (Fig. 6). Diagrams of P-waves are obtained by rotating the S-wave diagram by 45° . These diagrams are qualitatively close to diagrams from a double force couple without a moment in an elastic isotropic medium (White, 1983).

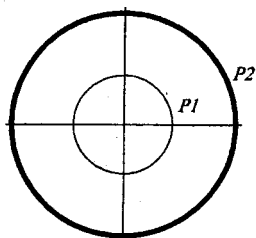


Fig. 5. Radiation patterns of source N 9 (longitudinal discontinuity of displacement in the fluid phase).

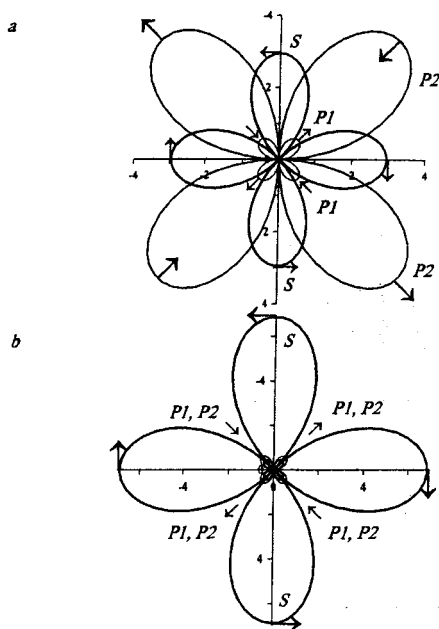


Fig. 6. Radiation patterns of source N 10 (transverse discontinuity of displacement in the solid phase).