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EFFECTIVE MODELS OF THINLY-STRATIFIED MEDIA CONTAINING POROUS, ELASTIC AND FLUID LAYERS

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ABSTRACT

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It is recognized that sedimentary rocks are often finely layered. If we consider them as an elastic medium, it results in an effective medium, which is also elastic, but anisotropic. We investigate the cases when the alternating constituents themselves are described as a poroelastic Biot medium or a mixed Biot-elastic or Biot-fluid medium. We show that in this case the Biot medium needs some generalization concerning density parameters. In other words, such interlayering causes all densities, including global and fluid densities, to be tensors. However, this generalization complicates the expressions and computations for the phase and group velocities only slightly.

The case of alternating solid and porous layers is considered as a special case of the porous-porous medium. The presence of solid layers prevents relative motion of fluid and solid across a lamination and, as a result, gives a triangular-shaped front of the "slow" longitudinal wave. This front may exist in different forms, which are more complex than similar fronts in the effective model of a stratified solid-fluid medium.

Demonstrations of wave fronts for certain parameters explain their possible behavior in all considered models.

KEY WORDS: effective model, thinly-stratified, porous Biot, elastic anisotropy, density anisotropy, wave fronts.

INTRODUCTION

The origin of seismic anisotropy is primarily due to fine layering of mostly isotropic constituents. This fine heterogeneity is clearly seen on acoustic and density log curves. Fortunately, in the case of elastic constituents an effective medium can also be regarded as a conventional elastic medium.

Porous media are assumed to be well described by the Biot model. What will happen if we consider a stack of thin poroelastic layers? Or a stack of alternating porous and solid or fluid layers? Will the effective medium stay within the frame of the classical Biot model? These questions are of practical interest for describing wave propagation in thinly layered porous rocks which have significant seismic and tremendous filtration anisotropy (Schoenberg, 1991, 1996). In fact, the porosity of reservoir rocks is often determined by isometric pores, while transport properties and seismic anisotropy are due to fracture systems which may be represented by thin layers with small elastic moduli (Schoenberg, 1991, 1996; Hsu and Schoenberg, 1993). In order to account for all these effects, it is necessary to learn how to average media with alternating porous layers. If we do this, we can make numerous transitions assuming one of the layers as thin and soft or impermeable or highly permeable etc. In this paper, we solve these basic problems by constructing three models of stratified porous-porous, porous-solid, and porous-fluid media.

STRATIFIED POROELASTIC MEDIUM

For simplicity, we consider the periodic media shown in Fig. 1 in two dimensions, each period containing two layers. In the first case, both layers are isotropic poroelastic Biot media. They are described by the eqs. (Biot, 1962)

$$\begin{aligned}\tau_{xx} &= P(\partial u_x / \partial x) + F(\partial u_z / \partial z) + M[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\ \tau_{zz} &= F(\partial u_x / \partial x) + P(\partial u_z / \partial z) + M[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\ -p &= M(\partial u_x / \partial x) + M(\partial u_z / \partial z) + R[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\ \tau_{xz} &= L[(\partial u_x / \partial z) + (\partial u_z / \partial x)] ,\end{aligned}\quad (1)$$

$$\begin{aligned}(\partial \tau_{xx} / \partial x) + (\partial \tau_{xz} / \partial z) &= \bar{\rho} \ddot{u}_x + \rho_f \ddot{w}_x , \\ (\partial \tau_{xz} / \partial x) + (\partial \tau_{zz} / \partial z) &= \bar{\rho} \ddot{u}_z + \rho_f \ddot{w}_z , \\ -\partial p / \partial x &= \rho_f \ddot{u}_x + m \ddot{w}_x , \\ -\partial p / \partial z &= \rho_f \ddot{u}_z + m \ddot{w}_z ,\end{aligned}\quad (2)$$

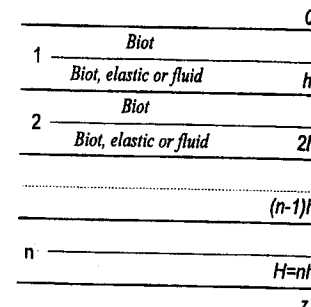


Fig. 1. Schematic representation of the periodic stack of layers under consideration with total thickness H and number of periods n .

where u_x, u_z = displacements in the solid phase, w_x, w_z = displacements of fluid relative to skeleton times porosity, $\tau_{xx}, \tau_{zz}, \tau_{xz}$ = total bulk stresses in the porous medium, p = fluid pressure, ρ = volume-averaged (global) density, ρ_f = fluid density, $m = \rho_f \alpha / \epsilon$, α = tortuosity of pore space, and ϵ = porosity. The constituents in the period have the portions θ_1 and θ_2 ($\theta_1 + \theta_2 = 1$). This medium, consisting in total of n periods, occupies the region from 0 to $H = nh$ along the vertical z -axis. Boundary conditions between the layers are welded contacts with free flow across the interfaces

$$\begin{aligned}u_{x1} &= u_{x2} , & u_{z1} &= u_{z2} , & w_{z1} &= w_{z2} , \\ \tau_{zz1} &= \tau_{zz2} , & \tau_{xz1} &= \tau_{xz2} , & p_1 &= p_2 .\end{aligned}\quad (3)$$

If we take into account conditions (3) and define some source, we may immediately construct an exact wave field in the stack of layers by means of the matrix method. This field accounts for all reflections and refractions at all the boundaries. In the expressions for the field we can also make a transition when the number of layers $n \rightarrow \infty$ and the period thickness $h \rightarrow 0$, whereas the total thickness of the stack is kept constant: $H = nh = \text{const}$. After this transition, we obtain new equations for some homogeneous, but anisotropic medium which is called the "effective model" (Molotkov, 1979).

In this paper, we use a more straightforward approach suggested by Molotkov (1992) and called a "simplified method of matrix averaging". Among eight variables entering eqs. (1) and (2) six are continuous, while w_x and τ_{xx} may have jumps at the interfaces. We first express derivatives of these continuous variables with respect to z through these variables themselves and their derivatives with respect to x and t . This yields the equations

$$\begin{aligned}
\partial u_x / \partial z &= -\partial u_z / \partial x + (1/L)\tau_{xz} , \\
\partial u_z / \partial z &= -[(FR - M^2)/(PR - M^2)](\partial u_x / \partial x) + [R/(PR - M^2)]\tau_{zz} + [M(PR - M^2)p , \\
\partial^3 w_z / \partial z \partial t^2 &= [(\rho_f/m) - \{2LM/(PR - M^2)\}](\partial^3 u_x / \partial x \partial t^2) - [M/(PR - M^2)](\partial^2 \tau_{zz} / \partial t^2) \\
&+ (1/m)(\partial^2 p / \partial x^2) - [P/(PR - M^2)](\partial^2 p / \partial t^2) , \\
\partial \tau_{xz} / \partial z &= [(m\bar{\rho} - \rho_f^2)/m](\partial^2 u_x / \partial t^2) - [\Delta/(PR - M^2)](\partial^2 u_x / \partial x^2) \\
&- [(FR - M^2)/(PR - M^2)](\partial \tau_{zz} / \partial x) - [(\rho_f/m) - 2LM/(PR - M^2)](\partial p / \partial x) , \\
\partial \tau_{zz} / \partial z &= \bar{\rho}(\partial^2 u_z / \partial t^2) + \rho_f(\partial^2 w_z / \partial t^2) - (\partial \tau_{xz} / \partial x) , \\
-\partial p / \partial z &= \rho_f(\partial^2 u_x / \partial t^2) + m(\partial^2 w_z / \partial t^2) ,
\end{aligned}
\tag{4}$$

where Δ is a determinant of the third order of Hooke's law matrix:

$$\Delta = P^2R + 2FM^2 - 2PM^2 - RF^2 . \tag{5}$$

We also should add two more equations expressing the discontinuous variables w_x and τ_{xx} through continuous variables

$$\begin{aligned}
\partial^2 w_x / \partial t^2 &= -(\rho_f/m)(\partial^2 u_x / \partial t^2) - (1/m)(\partial p / \partial x) , \\
\tau_{xx} &= [\Delta/(PR - M^2)](\partial u_x / \partial x) + [(FR - M^2)/(PR - M^2)]\tau_{zz} - [2LM/(PR - M^2)]p .
\end{aligned}
\tag{6}$$

After this rearrangement, averaging of the layered medium reduces to simple volume-averaging of derivatives of all continuous variables in the direction normal to the interfaces, which is the z -axis in this paper:

$$\begin{aligned}
\partial u_x / \partial z &= (\partial u_{x1} / \partial z)\theta_1 + (\partial u_{x2} / \partial z)\theta_2 , \\
\partial u_z / \partial z &= (\partial u_{z1} / \partial z)\theta_1 + (\partial u_{z2} / \partial z)\theta_2 , \\
\partial w_z / \partial z &= (\partial w_{z1} / \partial z)\theta_1 + (\partial w_{z2} / \partial z)\theta_2 , \\
\partial p / \partial z &= (\partial p_1 / \partial z)\theta_1 + (\partial p_2 / \partial z)\theta_2 , \\
\partial \tau_{xz} / \partial z &= (\partial \tau_{xz1} / \partial z)\theta_1 + (\partial \tau_{xz2} / \partial z)\theta_2 , \\
\partial \tau_{zz} / \partial z &= (\partial \tau_{zz1} / \partial z)\theta_1 + (\partial \tau_{zz2} / \partial z)\theta_2 .
\end{aligned}
\tag{7}$$

In eqs. (7) and further, the index "1" refers to the first layer in the period, whereas the index "2" refers to the second layer. Averaged derivatives

have no indices. This averaging is strictly proved by using the above-mentioned transition in the full wave field (Molotkov, 1992). Discontinuous variables are averaged by the following manner

$$w_x = w_{x1}\theta_1 + w_{x2}\theta_2 , \quad \tau_{xx} = \tau_{xx1}\theta_1 + \tau_{xx2}\theta_2 . \tag{8}$$

These formulas do not follow from the transition. However, they are quite useful to write because the resulting equations depend on the variables w_{x1} , w_{x2} , τ_{xx1} and τ_{xx2} only through w_x and τ_{xx} . Moreover, the introduced quantities have a physical meaning of period-averaged displacement and stress. This averaging is similar to averaging t_{xx} in conventional elasticity (Molotkov, 1979, 1992).

After implementing the scheme (7), we find the same eqs. as those of (4), but with all the coefficients replaced by their period- or volume-averaged values. For example, $(FR - M^2)/(PR - M^2)$ in the second line of eq. (4) is replaced by $(\overline{FR - M^2})/(\overline{PR - M^2})$, and ρ_f/m in the third line of eq. (4) is replaced by $(\overline{\rho_f/m})$; the bar above the value means volume averaging according to the formula

$$\bar{s} = \theta_1 s_1 + \theta_2 s_2 . \tag{9}$$

Transforming these equations back to the form of eqs. (1) and (2) in the same way as we have used for a direct transformation, we obtain the relations

$$\begin{aligned}
\tau_{xx} &= \bar{P}(\partial u_x / \partial x) + \bar{F}(\partial u_z / \partial z) + \bar{M}[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\
\tau_{zz} &= \bar{F}(\partial u_x / \partial x) + \bar{C}(\partial u_z / \partial z) + \bar{Q}[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\
-p &= \bar{M}(\partial u_x / \partial x) + \bar{Q}(\partial u_z / \partial z) + \bar{R}[(\partial w_x / \partial x) + (\partial w_z / \partial z)] , \\
\tau_{xz} &= \bar{L}[(\partial u_x / \partial z) + (\partial u_z / \partial x)] ,
\end{aligned}
\tag{10}$$

$$\begin{aligned}
(\partial \tau_{xx} / \partial x) + (\partial \tau_{xz} / \partial z) &= \bar{\rho}_x \ddot{u}_x + \rho_{fx} \ddot{w}_x , \\
(\partial \tau_{xz} / \partial x) + (\partial \tau_{zz} / \partial z) &= \bar{\rho}_z \ddot{u}_z + \rho_{fz} \ddot{w}_z , \\
-\partial p / \partial x &= \rho_{fx} \ddot{u}_x + m_x \ddot{w}_x , \quad -\partial p / \partial z = \rho_{fz} \ddot{u}_z + m_z \ddot{w}_z ,
\end{aligned}
\tag{11}$$

where

$$\begin{aligned}
\bar{\rho}_x &= \bar{\rho}, \bar{\rho}_x = ((\bar{\rho}m - \rho_f^2)/m) + (\overline{\rho_f/m})^2(\overline{1/m})^{-1} , \\
\bar{\rho}_{fx} &= (\overline{\rho_f/m})(\overline{1/m})^{-1} , \quad \bar{\rho}_{fz} = \bar{\rho}_f, m_x = (\overline{1/m})^{-1} , \quad m_z = \bar{m} ,
\end{aligned}
\tag{12}$$

$$\begin{aligned}
\tilde{P} &= (\overline{(FR - M^2)/d})\tilde{F} + (\overline{(2LM/d)})\tilde{M} + (\overline{(D/d)}) , \\
\tilde{F} &= (1/f)\{(\overline{(P/d)})(\overline{(FR - M^2)/d}) + (\overline{(M/d)})(\overline{(2LM/d)})\} , \\
\tilde{M} &= (1/f)\{(\overline{(M/d)})(\overline{(FR - M^2)/d}) + (\overline{(R/d)})(\overline{(2LM/d)})\} , \\
\tilde{C} &= (1/f)(\overline{(P/d)}) , \quad \tilde{Q} = (1/f)(\overline{(M/d)}) , \quad \tilde{R} = (1/f)(\overline{(R/d)}) , \quad \tilde{L} = (\overline{(1/L)})^{-1} , \\
f &= (\overline{(P/d)})(\overline{(R/d)}) - (\overline{(M/d)})^2 , \quad d = PR - M^2 , \quad D = \begin{vmatrix} P & F & M \\ F & P & M \\ M & M & R \end{vmatrix} .
\end{aligned} \tag{13}$$

These equations correspond to a homogeneous two-phase medium which is a generalization of the transversely isotropic Biot model. This generalization concerns only eqs. (2) of the continuous medium. Whereas in the transversely isotropic Biot model only density m may be a tensor, in eqs. (11) the global (ρ) and fluid (ρ_f) densities also become tensors and, therefore, instead of 4 densities as in the conventional Biot model, we have 6 densities in the generalized transversely isotropic Biot model. These two additional parameters are due to different averaging of densities along and across layers. Such a difference was first noted by Molotkov (1979) during averaging of a stratified fluid-fluid medium where anisotropy is solely due to the tensor (anisotropic) density.

It may be shown that both the kinetic and the potential energy of the effective model are positive, definite quadratic forms if we use initially stable constituents for averaging (with positive, definite kinetic and potential energy densities).

In the general case, $\bar{\rho}_x \neq \bar{\rho}_z$ and $\rho_{fx} \neq \rho_{fz}$. However, in the special case when the fluid density is constant in one period ($\rho_{fi} = \rho_{fz}$), then $\bar{\rho}_x = \bar{\rho}_z$, $\rho_{fx} = \rho_{fz}$, and the effective model becomes an ordinary transversely isotropic Biot medium. This is an important fact which justifies application of the original anisotropic Biot model for fine-layered porous sediments.

If we exclude the consideration of SH-waves, then in the generalized, transversely isotropic Biot model three waves propagate and their velocities along the axes are given by

$$\begin{aligned}
v_{1,2}^2 &= [\pm \sqrt{\{(\tilde{R}\bar{\rho}_x + \tilde{P}m_x - 2\tilde{M}\rho_{fx})^2 - 4(\bar{\rho}_xm_x - \rho_{fx}^2)(\tilde{P}\tilde{R} - \tilde{M}^2)\}} \\
&\quad + \tilde{R}\bar{\rho}_x + \tilde{P}m_x - 2\tilde{M}\rho_{fx}]/2(\bar{\rho}_xm_x - \rho_{fx}^2) , \\
v_3^2 &= \tilde{L}m_z/(\bar{\rho}_zm_z - \rho_{fz}^2) ,
\end{aligned}$$

$$\begin{aligned}
v_{4,5}^2 &= [\pm \sqrt{\{(\tilde{R}\bar{\rho}_z + \tilde{C}m_z - 2\tilde{Q}\rho_{fz})^2 - 4(\bar{\rho}_zm_z - \rho_{fz}^2)(\tilde{C}\tilde{R} - \tilde{Q}^2)\}} \\
&\quad + \tilde{R}\bar{\rho}_z + \tilde{C}m_z - 2\tilde{Q}\rho_{fz}]/2(\bar{\rho}_zm_z - \rho_{fz}^2) , \\
v_6^2 &= \tilde{L}m_x/(\bar{\rho}_xm_x - \rho_{fx}^2) ,
\end{aligned} \tag{14}$$

where the first three velocities are fast longitudinal (P1), "slow" longitudinal (P2) and shear (SV) velocities along the x-axis; the remaining three velocities are the same for the z-axis. Note that the expressions for the P1 and P2-wave velocities along the x-axis contain only x-ed densities and along the z-axis contain only z-ed densities, whereas for the SV-wave they are interchanged. Thus for the principal axes, all velocities are affected only by densities and pore space properties in the direction of particle displacement. Second and third wave fronts for certain parameters may have loops, which result in additional intersections with the coordinate axes. These additional points of intersection correspond to singularities of solutions of the Christoffel equation. Examples of wavefronts from a point source (group velocity curves) are given in Fig. 2. ($\tilde{P} = 50$ GPa, $\tilde{C} = 20$ GPa, $\tilde{F} = 8$ GPa, $\tilde{M} = \tilde{Q} = 10$ GPa, $\tilde{R} = 10$ GPa, $\tilde{L} = 4$ GPa, $\bar{\rho}_x = 2.36$ g/cm³, $\bar{\rho}_z = 3.54$ g/cm³, $\rho_{fx} = 1$ g/cm³, $\rho_{fz} = 1.5$ g/cm³, $m_x = 15$ g/cm³, $m_z = 45$ g/cm³).

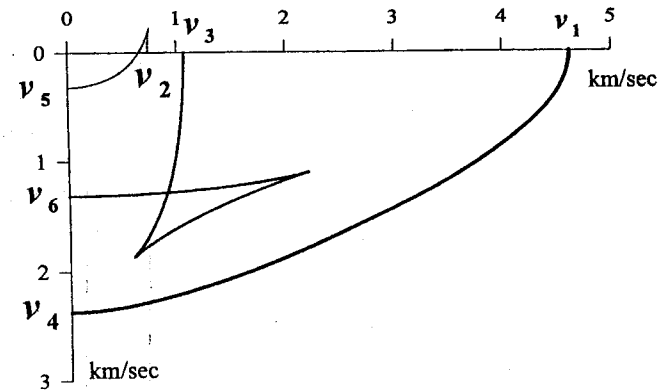


Fig. 2. Wave fronts from a point source (group velocity curves) in the generalized transversely isotropic Biot model which has tensoric global, fluid and tortuous density (parameters are given in the text).

MEDIUM OF ALTERNATING POROUS AND IMPERMEABLE SOLID LAYERS

For a second model we assume the same geometry (Fig. 1) and notations as for the first model, except that now the second layers in the period are elastic and impermeable with Lamé parameters λ and μ and density ρ . Boundary conditions correspond to a welded contact:

$$u_{x1} = u_{x2}, \quad u_{z1} = u_{z2}, \quad w_{z1} = 0, \quad \tau_{zx1} = \tau_{zx2}, \quad \tau_{xz1} = \tau_{xz2}. \quad (15)$$

To obtain the effective model and all parameters in this case, we use the fact that the Biot model is a generalization of solid and fluid media. Thus for the second layer we make a transition to the case when the porosity tends to $\epsilon \rightarrow 0$. With this transition, other parameters behave in the following manner:

$$P \rightarrow \lambda + 2\mu, \quad F \rightarrow \lambda, \quad L \rightarrow \mu, \quad R \rightarrow \infty, \quad m \rightarrow \infty. \quad (16)$$

These limiting values of elastic moduli may easily be verified by using the same transition ($\epsilon \rightarrow 0$) in the elastic parameters of the Biot model expressed through the bulk moduli of the fluid, solid material composing the porous frame, the porous frame as a whole and the shear modulus and porosity according to the well-known formulas given, for example, by Geertsma and Smith (1961) or Stoll (1974). Density m goes to infinity because it is inversely proportional to the porosity: $m = \rho_f \alpha / \epsilon$, whereas $\alpha \geq 1$. Such behavior of m is also consistent with the relation of Berryman (1981) who suggested that for low porosity, tortuosity grows inversely proportionally to porosity. It is important that the value of the modulus M in eqs. (1) is finite; this is also in agreement with the relations of Geertsma and Smith (1961) or Stoll (1974).

After substitution of the relations (16) into eqs. (1) and (2), the first, second and fourth eqs. of (1) together with the two first eqs. of (2) describe an ordinary isotropic elastic medium. Relative displacement $w = \epsilon(u^{(1)} - u^{(2)})$ is set at zero, as $\epsilon \rightarrow 0$. The third eq. (1) together with the third and fourth eqs. of (2) are rejected as uncertain.

Hooke's law for the effective model of a stratified porous-solid medium is expressed by eqs. (10) with the effective parameters again given by formulas (13) if, according to relations (16), we understand the averaged combinations with bars in the following way

$$\begin{aligned} \overline{(FR - M^2/d)} &= \{(FR - M^2)/d\}\theta_1 + \{\lambda/(\lambda + 2\mu)\}\theta_2, \\ \overline{(2LM/d)} &= (2LM/d\bar{M})\theta_1, \quad \overline{(D/d)} = (D/d)\theta_1 + \{4\mu(\lambda + \mu)/(\lambda + 2\mu)\}\theta_2, \\ \overline{(P/d)} &= (P/d)\theta_1, \quad \overline{(M/d)} = (M/d)\theta_1, \end{aligned}$$

$$\overline{(R/d)} = (R/d)\theta_1 + \{1/(\lambda + 2\mu)\}\theta_2,$$

$$\overline{(1/L)} = (1/L)\theta_1 + (1/\mu)\theta_2. \quad (17)$$

Here P , F , M , R and L are the intrinsic Biot moduli of the first, porous layer with portion θ_1 , whereas λ and μ are the Lamé parameters of the second, solid layer with portion θ_2 . If we use eqs. (12) for densities and take into account that the density m in the solid layer tends to infinity, we find

$$\bar{\rho} = \bar{\rho}_x = \bar{\rho}_z = \theta_1 \bar{\rho} + \theta_2 \rho, \quad \rho_{fx} = \rho_f, \quad m_x = m/\theta_1, \quad m_z = \infty, \quad (18)$$

where $\bar{\rho}$, ρ_f and m are the intrinsic densities of the first layer in the period (Biot layer); ρ is the density of the second layer in the period (solid layer). Note that ρ_{fx} is uncertain and m_z is infinite. This means that we should first set w_z equal to zero in order to keep the kinetic energy finite, and secondly reject the last equation of (11) as uncertain. Reducing the number of equations is consistent with the loss of one degree of freedom because the variable w_z is now zero in the whole medium. The equations of a continuous medium are then written as

$$\begin{aligned} \partial \tau_{xx} / \partial x + \partial \tau_{xz} / \partial z &= \bar{\rho} \ddot{u}_x + \rho_f \ddot{w}_x, \quad \partial \tau_{xz} / \partial x + \partial \tau_{zz} / \partial z = \bar{\rho} \ddot{u}_z, \\ -\partial p / \partial x &= \rho_f \ddot{u}_x + m_x \ddot{w}_x, \end{aligned} \quad (19)$$

According to eqs. (10) and (19), the effective model is a special case of the transversely isotropic Biot model which has

$$m_z = \infty, \quad w_z = 0, \quad (20)$$

and the last equation, containing $\partial p / \partial z$, is rejected as uncertain.

When we take these conditions into account, velocities v_1 , v_2 and v_6 will still be expressed by the formulas (14), but the other velocities will be given by

$$v_3^2 = \bar{L} / \bar{\rho}, \quad v_4^2 = \bar{C} / \bar{\rho}, \quad v_5^2 = 0. \quad (21)$$

From formulas (21), one can see that in the second model only two waves propagate along the z -axis. This means that one wave should have a front which starts and finishes on the x -axis. This wave has an additional velocity along the x -axis:

$$v_7^2 = (\bar{C}\bar{R} - \bar{Q}^2) / (\bar{C}m_x). \quad (22)$$

Analysis of eq. (22) shows that this velocity is a velocity in a plate made up of the transversely isotropic Biot medium and covered by a thin impermeable

film so that the fluid cannot move in and out of the pores. Some typical wave fronts (group velocity curves) are given in Figs. 3 and 4, where porosity $\epsilon = 0.2$ and $\bar{\rho} = 2.36 \text{ g/cm}^3$, $\rho_f = 1 \text{ g/cm}^3$, $m_x = 15 \text{ g/cm}^3$. Elastic parameters for Fig. 3 are $\bar{P} = 50 \text{ GPa}$, $\bar{C} = 20 \text{ GPa}$, $\bar{F} = 8 \text{ GPa}$, $\bar{Q} = 12 \text{ GPa}$, $\bar{M} = 10 \text{ GPa}$, $\bar{R} = 8 \text{ GPa}$, $\bar{L} = 4 \text{ GPa}$, while for Fig. 4, $\bar{P} = 33.8 \text{ GPa}$, $\bar{C} = 15 \text{ GPa}$, $\bar{F} = 6.9 \text{ GPa}$, $\bar{Q} = \bar{M} = 7.5 \text{ GPa}$, $\bar{R} = 30 \text{ GPa}$, $\bar{L} = 1 \text{ GPa}$. In Fig. 3, the triangular-shaped wavefront corresponds to the P2-wave (second "slow" longitudinal wave) and it may be obtained as a limiting case of the inner front from Fig. 2 when $v_5 \rightarrow 0$. In Fig. 4 a more interesting situation is shown. The second front starts with velocity v_2 and pure longitudinal polarization and finishes at velocity v_6 with pure shear polarization. On the inner triangular-shaped front, we have the opposite situation. Such transitions in the Biot model between P2- and SV-wave were first noted and explained by Bakulin and Molotkov (1995, 1996).

Finally, we should note that independent matrix averaging of a porous-solid medium confirms all given results obtained by using a porous-porous model and considering the solid layer as a limiting case of the Biot layer ($\epsilon \rightarrow 0$).

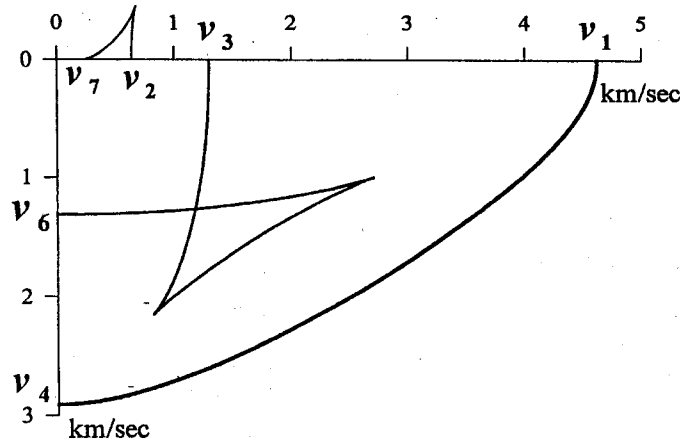


Fig. 3. Wave fronts from a point source (group velocity curves) for a particular transversely isotropic Biot model which is an effective model of the stratified porous-solid medium in this figure (parameters are given in the text).

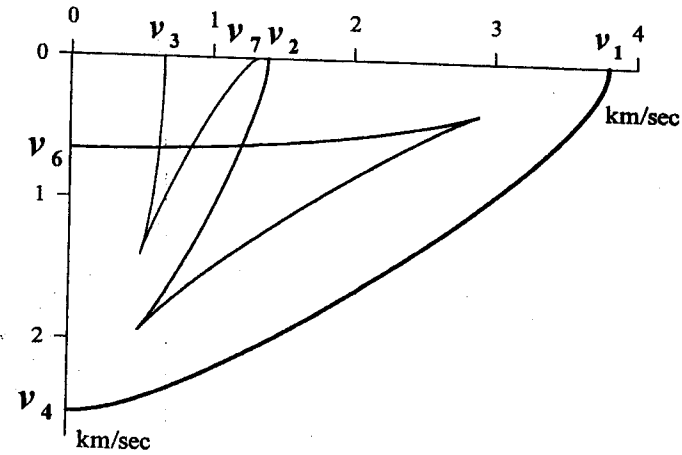


Fig. 4. Wave fronts from a point source (group velocity curves) for a particular transversely isotropic Biot model, which is an effective model of the stratified porous-solid medium (parameters are given in the text). Note that velocities v_2 and v_7 correspond to the P2-wave, while v_6 and v_3 belong to SV-wave; thus, transitions of P2 to SV-waves and of SV to P2-waves take place in the second and third wave fronts, respectively.

ALTERNATING POROELASTIC AND FLUID LAYERS

The third effective model describing a stratified fluid-porous medium is probably not so important for practical applications, but very important for understanding wave propagation in porous media with open continuous fractures of very simple geometry. Unfortunately, we have not yet discovered how to obtain this model from the general model of porous-porous media directly as we did in the previous section for a porous-solid medium. However, the described method of simplified matrix averaging is still applicable for such layers. If we assume the boundary conditions of free flow

$$u_{z1} + w_{z1} = u_{z2}, \quad \tau_{xz1} = 0, \quad \tau_{zz1} = -p_1 = -p_2, \quad (23)$$

then we find that the effective model is a special case of the generalized transversely isotropic Biot model, eqs. (10) and (11), when $\bar{L} = 0$, $\bar{F} = \bar{M}$, $\bar{C} = \bar{Q} = \bar{R}$, $m_z = \infty$ and $\bar{\rho}_z$ is arbitrary and finite. After these substitutions, the effective model in two dimensions is described by the equations

$$\begin{aligned}\tau_{xx} &= P(\partial u_x / \partial x) + F[(\partial u_z / \partial z) + (\partial w_x / \partial x)], \\ \tau_{zz} &= \tilde{F}(\partial u_x / \partial x) + \tilde{C}[(\partial u_z / \partial z) + (\partial w_x / \partial x)],\end{aligned}\quad (24)$$

$$\begin{aligned}\partial \tau_{xx} / \partial x &= \bar{\rho}_x (\partial^2 u_x / \partial t^2) + \bar{\rho}_t (\partial^2 w_x / \partial t^2), \\ \partial \tau_{zz} / \partial x &= \bar{\rho}_t (\partial^2 u_x / \partial t^2) + m_x (\partial^2 w_x / \partial t^2), \\ \partial \tau_{zz} / \partial z &= \bar{\rho}_z (\partial^2 u_z / \partial t^2).\end{aligned}\quad (25)$$

The first front from a point source in this model corresponds to the fast longitudinal wave with velocities v_1 and v_4 along the x - and z -axes. The inner front of the "slow" longitudinal wave starts and finishes on the x -axis and has two velocities, v_2 and v_8 . Expressions for v_1 and v_2 are still the same as in formulas (14) whereas v_4 and v_8 are calculated by

$$v_4^2 = \tilde{C} / \bar{\rho}_z, \quad v_8^2 = m_x (\tilde{P} \tilde{R} - \tilde{M}^2) / [\tilde{R}(\bar{\rho}_x m_x - \bar{\rho}_t^2)]. \quad (26)$$

Additional analysis yields that v_8 is a velocity in a plate made up of the transversely isotropic Biot medium and having free boundaries.

For all parameters, the velocities along the axes satisfy the inequalities

$$v_2 \leq v_8 \leq v_1. \quad (27)$$

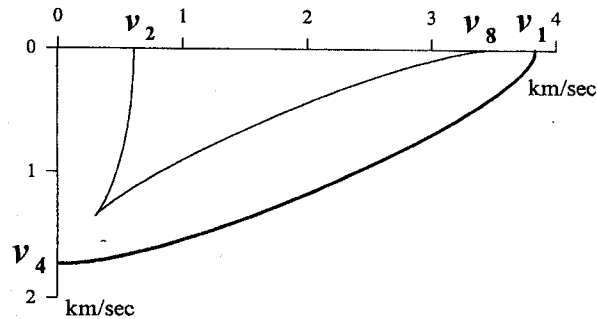


Fig. 5. Wave fronts from a point source (group velocity curves) in the generalized transversely isotropic Biot model which is an effective model of the stratified porous-fluid medium.

These wavefronts (group velocity curves) are shown in Fig. 5 and may not have the more complicated (double) loops, which could exist on fronts in an ordinary transversely isotropic Biot model (Molotkov and Bakulin, 1995, 1996).

If we set the tortuosity in this model $\alpha_x = 1$ or $m_x = \rho_t / \epsilon$, where ϵ is the total porosity of the layered medium, then the eqs. (24) and (25) will be the same as the equations of the effective model of the layered fluid-solid medium (Molotkov and Bakulin, 1995), which is a special case of the ordinary, transversely isotropic Biot model (Molotkov and Bakulin, 1995, 1996). The wavefronts in both models are qualitatively the same and are given in Fig. 5.

CONCLUSIONS

We have studied wave propagation in thinly-stratified media with alternating porous, elastic and fluid layers and generalized Backus averaging for this type of medium. The constructed effective model of a stratified, poroelastic medium is shown to be a generalization of the Biot model. This means that the total and fluid densities are different for x - and z -directions. However, this generalization does not strongly complicate the expressions for velocities along the axes, which are similar to the ordinary transversely isotropic Biot medium.

The effective model of alternating porous and non-porous layers is proved to be a special case of the ordinary transversely isotropic Biot model with infinite tortuosity across the layers. The triangular-shaped wave front propagating in this model as a rule corresponds to a second "slow" P2-wave, whose velocity is zero across the lamination.

In the case of alternating porous and fluid layers, the effective model is identical with the generalized transversely isotropic Biot model. This model and its wave fronts are very close to the effective model of alternating solid-fluid layers. It also has a triangular second wave front corresponding to the "slow" longitudinal wave.

In forthcoming publications we plan to numerically investigate the ranges of anisotropy of all parameters caused by interlayering of porous and solid constituents for some real reservoir rocks.

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