EFFECT OF STACKING ON MULTIPLICATIVE NOISE CAUSED BY SMALL-SCALE SCATTERING

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Summary

We propose a simple statistical model of random multiplicative noise to describe wavefield distortions caused by small-scale near-surface scattering. We specify two types of multiplicative noise: random phase perturbation and random time shifts (residual statics). We investigate the transformation of amplitude and phase of locally stacked data and demonstrate that the combined action of phase perturbations and residual statics could explain what is observed on real land seismic data from the desert environment. Phase perturbations jumble the coherency of all events and lead to severe amplitude loss during stacking. Residual statics creates additional amplitude loss, progressively increasing with frequency. Remarkably, the phase of the local stack delivers an unbiased estimate of the clean signal phase, justifying the previosuly proposed phase substitution method and paving the way to mitigating multiplicative noise in seismic processing. We also reveal a clear link of the new model to optical and ultrasonic speckle noise, opening opportunities for exploring new methods from other domains.

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Introduction

Seismic data often exhibit highly distorted and chaotic prestack reflections even after processing in areas with complex near surface. Despite such complexity, local stacking methods are typically able to recover clear reflections (Bakulin et al., 2020a, 2020b). However, the absolute level of amplitude spectra after such stacking experiences a substantial decline across the entire frequency band, reaching -10-25 dB or more. In addition, a significant and progressive loss of higher frequencies is observed. Field observations are blamed on "complex near surface", yet no simple models explain such behavior. Inspired by studies of optical and ultrasonic speckle noise (Goodman, 2007), we put forward a simple statistical model that can reproduce experimental observations and pave the way to mitigate near-surface scattering noise.

Field observations from the desert environment

A typical example of NMO-corrected common-midpoint (CMP) gather is shown in Figure 1a. The input data have already been processed through a standard land flow, yet the prestack signal remains highly distorted. There are no identifiable reflections in the gather. In addition, the entire gather appears chaotic from top to bottom and from small offset to large. Figure 1b shows the same gather but after local stacking using Nonlinear Beamforming or NLBF (Bakulin et al., 2020a). NLBF uses local ensembles of around 1000 neighboring traces for the local summation. After the enhancement, the reflections become visible in the entire offset range. However, reflection events become overly smoothed while there are significant changes in the spectra (Figure 1c). If we are to summarize main observations from similar datasets, they become:

A. After typical time processing, prestack reflections remain chaotic with low coherency. In addition, such reflection disorder is not confined in time-space but instead spread over the entire gather.
B. After local stacking, reflections become trackable and coherent. However, the absolute level of amplitude spectra experiences a strong shift or bias downward (-15-25 dB) across all frequencies.
C. There is escalating loss of higher frequencies after stacking.

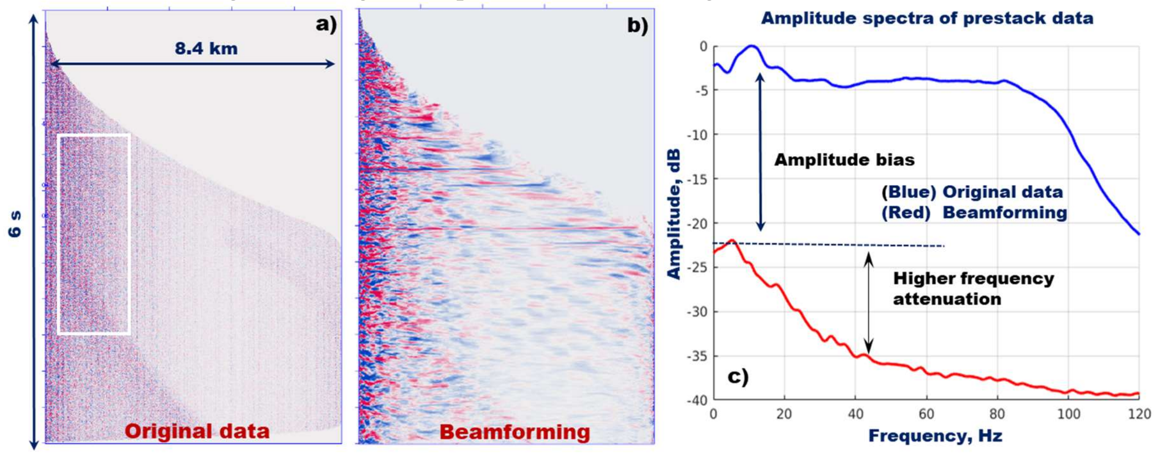

Figure 1. *CMP gather (after NMO) extracted from the land seismic dataset: (a) after standard processing; (b) the same gather, but after nonlinear beamforming; (c) comparison of amplitude spectra computed on data from (a) and (b) (calculated inside the white box).*

Stacking in the presence of residual statics acts as a low-pass filter (Berni and Rover, 1989) with low frequencies barely reduced. However, we observe significant attenuation (-15 to -25 dB) of amplitudes below 20 Hz. This study puts forward a mathematical model for multiplicative noise that consistently explains all field observations. We invoke multiplicative noise of two types. The first type is described by random phase perturbations varying with frequency. The second type is represented by random time-delays (residual statics) between channels. Phase perturbations themselves lead to a severe loss of coherency on prestack gathers and produce a strong downward bias of broadband amplitudes after stacking. Time delays can explain the progressive loss of higher frequencies. Alternatively, frequency-dependent phase perturbations can also similarly describe the amplitude roll-off. Near-surface scattering layers can physically generate both types of multiplicative noise with small-to-medium-scale geological heterogeneities.

Mathematical model for local stacking ensemble

We assume that microscopic details of the near surface cannot be deterministically recovered (Goodman, 2007). This is often the case for medium and small-scale heterogeneities. Therefore, we invoke a linear system with random parameters to describe seismic traces $x_k(t)$ of the local ensemble affected by the propagation through the complex near surface. The signal model may be written as

$$x_k(t) = r_k(t) * s(t) + n_k(t), \quad (1)$$

whereas "*" denotes convolution, and signal $s(t)$ is assumed identical on all channels inside the local ensemble. Suppose local ensemble is drawn from traces sharing similar spatial locations. In that case, such an assumption is approximately valid (see Figures 1a,b, and *Observation B*). In contrast, random multiplicative noise $r_k(t)$ is different for each channel $k=1,...,K$ (K is a number of the local ensemble). In addition, $n_k(t)$ is random additive noise. The multiplicative noise model is universally accepted as a useful description of optical and ultrasonic speckle noise caused by small-scale rough surface or volumetric scattering (Goodman, 2007). However, it is not broadely utilized in seismic. In the Fourier domain (1) it can be written as:

$$X_k(\omega) = R_k(\omega)S(\omega) + N_k(\omega), \quad (2)$$

where X_k, R_k, S, N_k are Fourier transforms of the corresponding time-domain functions in (1). Additive noise $N_k(\omega)$ is uncorrelated between different channels and has zero mean, i.e. $E[N_k(\omega)] = 0$, where an E denotes mathematical expectation. In addition, signal and noise are assumed uncorrelated.
The local stack over an ensemble of traces is calculated as:

$$\bar{X}(\omega) = \frac{1}{K}\sum_{k=1}^{K} X_k(\omega). \quad (3)$$

A large ensemble of several hundreds of traces for local stacking provides a sufficient dataset to approximate the mathematical expectation of the described random process, mainly describing localized random seismic distortions due to small-scale near-surface scattering. But, first, let us examine its information content and properties. Mathematically this is expressed as:

$$\bar{X}(\omega) \approx E[X_k(\omega)] = E[R_k(\omega)S(\omega)] + E[N_k(\omega)] = S(\omega)\, E[R_k(\omega)]\,. \quad (4)$$

Equation (4) provides the theoretical prediction of how summation transforms amplitude and phase of the stacked result. Let us denote

$$\Phi(\omega) = E[R_k(\omega)]. \quad (5)$$

The first type of multiplicative noise is described as a random frequency-dependent phase fluctuations:

$$R_k(\omega) = e^{i\varphi_k(\omega)}. \quad (6)$$

We assume that random variables $\varphi_k(\omega)$ are independent for all channels and have the same probability density function $P(\omega; x)$ at a given frequency ω. One can write (5) as:

$$\Phi(\omega) = \int_{-\infty}^{\infty} P(\omega; x)e^{ix}\, dx. \quad (7)$$

Equation (7) leads to an important property that if the probability density function of the phase fluctuations of the multiplicative noise is even, i.e., $P(\omega; x) = P(\omega; -x)$ (implying that expected value is zero), then $\Phi(\omega)$ is real-valued (Goodman, 2007). Using notation (5), we may rewrite (4) as

$$E[X_k(\omega)] = |S(\omega)|e^{i\varphi_S(\omega)}\, \Phi(\omega)\,, \quad (8)$$

where $|S(\omega)|$, φ_S are amplitude and phase spectra of the clean signal. Without specifying any further microscopic details of the near surface, we can make the following conclusions. First, we observe the phase averaging or "cleanup" process where random symmetric phase fluctuations average out during stacking and lead either to signal phase (if $\Phi(\omega) > 0$), or flipped signal phase rotated by π (if $\Phi(\omega) < 0$). Second, real-valued $\Phi(\omega)$ denotes filtering loss factor for the signal amplitude spectra during stacking. To obtain quantitative results, let us derive a specific form of $\Phi(\omega)$ for the two types of noise: random phase perturbations and random time-shifts (residual statics), both having a normal distribution.
Type 1 multiplicative noise – Phase perturbation with normal distribution: If random variables $\boldsymbol{\varphi_k(\omega)}$ describing phase perturbations have the same normal (Gaussian) distribution with zero mean and standard deviation $\sigma(\omega)$, then mathematical expectation can be expressed as

$$E[X_k(\omega)] = |S(\omega)|e^{i\varphi_S(\omega)}e^{-\frac{\sigma^2(\omega)}{2}}. \quad (9)$$

While it was already inferred from the symmetry considerations, we stress again that the resulting phase spectrum after stacking in expression (9) is the same as the phase of the clean signal, i.e.

$$arg\{E[X_k(\omega)]\} = \varphi_S(\omega), \quad (10)$$

whereas a real-valued factor reduces amplitude compared to a clean signal. Amplitude reduction is identical across the entire band if standard deviations remain constant for all frequencies. In contrast, the loss of signal amplitude after stacking would progressively increase with frequency if we assume that standard deviation increases with frequency.

Another type of multiplicative noise can be represented by well-known residual statics.

Type 2 multiplicative noise – Residual statics with normal distribution: Residual statics, i.e. random time shifts τ_k between channels, can be recast as a specialized type of multiplicative noise:

$$R_k(\omega) = e^{-i\omega\tau_k}. \quad (11)$$

If residual statics τ_k has a normal distribution with zero mean and standard deviation σ, then we obtain the following expression for mathematical expectation of local stack:

$$E[X_k(\omega)] = |S(\omega)|e^{i\varphi_S(\omega)}e^{-\frac{\omega^2\sigma^2}{2}}. \quad (12)$$

Again, the phase spectrum of the stack (12) is the same as that of the clean signal. Amplitude experiences exponential loss with frequency. Berni and Roever (1989) obtained exponential amplitude loss analyzing intra-array residual statics without employing a multiplicative model.

Combination of Type 1 and Type 2: Let us consider a combination of both types of multiplicative noise (6) and (11) acting together. Assume that τ_k and $\varphi_k(\omega)$ are independent of each other and both random normally distributed with standard deviations σ_φ and σ_τ, then the mathematical expectation is given by:

$$E[X_k(\omega)] = |S(\omega)|e^{i\varphi_S(\omega)}e^{-\frac{\omega^2\sigma_\tau^2}{2}}e^{-\frac{\sigma_\varphi^2}{2}}. \quad (13)$$

The phase of mathematical expectation is equal to the clean signal phase. In contrast, the amplitude loss factor is a product of two terms – one caused by phase perturbations and another by residual statics.

A numerical example of local stacking with multiplicative noise

Figure 2a shows the results of the numerical simulations using land vibroseis Klauder wavelet. First, an ensemble of traces is generated using random realizations of multiplicative noise (Figures 2b,c) following a mathematical model (1). Then, we numerically stack traces from the ensemble (1000 traces as used in real data example) and compare with amplitude and phase predicted by theoretical equations for mathematical expectation above. Let us consider and contrast two cases: residual statics only ($\sigma_T =$ 4 ms) and the combined effect of residual statics ($\sigma_T = 4\ ms$) and phase perturbations ($\sigma_{Ph} = \pi/2$). Figure 3 shows the amplitude, phase, and time-domain representation of the stacked data versus the clean signal. Figure 3c shows how phase fluctuations can distort the input trace shown in green. In the time domain, stacked waveform experiences broadening (Figure 3c). The stacked phase agrees with the phase of the clean signal (Figure 3b). While residual statics leads to a progressive loss of higher frequencies, low frequencies are not affected (Figure 3a). In contrast, phase perturbations drop the amplitude level by ~12 dB. We observe good agreement with theory in all cases, thus validating the usefulness of statistical predictions for realistic local stacking scenarios with a limited number of traces. Comparing Figures 1c and 3a, we conclude that the combined effect of both types of multiplicative noise is required to explain experimental *Observations A-C*. Indeed, phase perturbations can explain *Observations A* and *B* since they lead to loss of coherency (Figure 2c) and severe amplitude bias after stacking (Figure 3a). Residual statics adds progressive loss of higher frequencies explaining *Observation C*.

Conclusions

We put forward a statistical multiplicative noise model describing the effect of near-surface scattering. Residual statics and phase perturbations are invoked as two main types of such random noise. We then establish fundamental properties of how multiplicative noise transforms while stacking. The first essential finding reveals that stacking produces an unbiased estimate of the clean signal phase using realistic, complex assumptions about noise. The second finding gives the mathematical relationship between frequency-dependent loss of amplitude during stacking and standard deviation of residual statics and phase perturbations. These findings serve as a theoretical justification for the previously proposed method of phase substitution (Bakulin et al., 2020b) and open the way to efficiently address multiplicative noise in seismic processing.

References

Bakulin, A., Silvestrov, I., Dmitriev, M., Neklyudov, D., Protasov, M., Gadylshin, K., 2020a, Nonlinear beamforming for enhancement of 3D prestack land seismic data: Geophysics, 85, 1MJ-Z13.

Bakulin, A., Neklyudov, D., and I. Silvestrov, 2020b, Prestack data enhancement with phase corrections in time-frequency domain guided by local multidimensional stacking: Geophysical Prospecting, 68, 1811-1818.

Berni, A.J., and W.L.Roever,1989, Field array performance: Theoretical study of spatially correlated variations in amplitude coupling and static shift and case study in the Paris Basin,Geophysics, 54, 451-459.

Goodman, J.W.,2007, Speckle Phenomena in Optics: Theory and Applications, Roberts & Company.

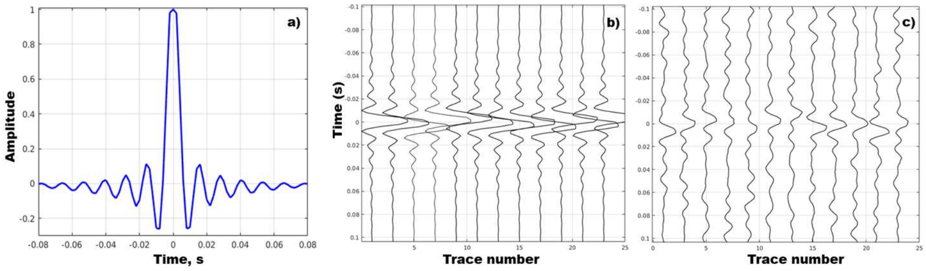

Figure 2. *Numerical simulations with multiplicative noise: (a) clean signal in time-domain is represented by Klauder wavelet or autocorrelation of the linear sweep 5-80 Hz with tapers; (b) traces with random timeshifts following normal distribution ($\sigma_T = 4\,ms$); (c) combination of normally distributed random phase perturbations ($\sigma_{Ph} = \pi/2$) and timeshifts ($\sigma_T = 4\,ms$). Each channel is obtained using a fixed clean signal (a) and random realizations of two types of multiplicative noises. While (b) has only a slight variation of arrival times, panel c) exhibits strong waveform distortion from channel to channel, reducing the coherency and hampering stacking.*

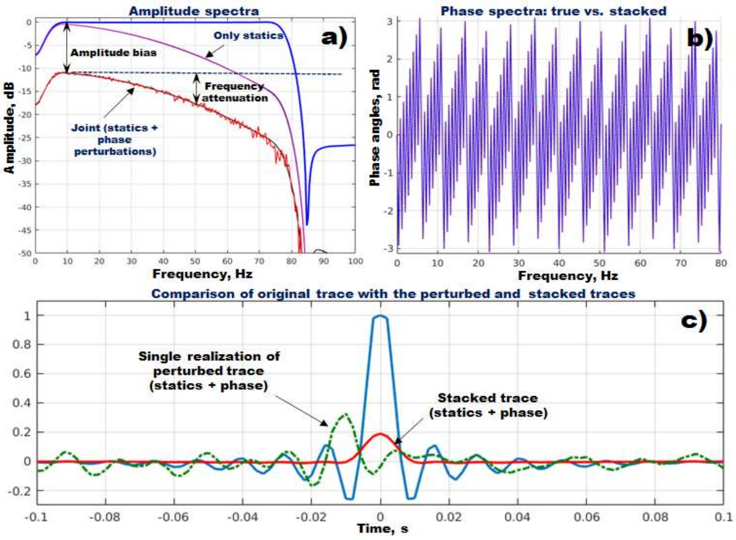

Figure 3. *Effect of multiplicative noise on stacked amplitude and phase spectra. The pink curve shows stacked results in the presence of residual statics only. Red curves show the joint effect of residual statics and phase perturbations. Both random values have a normal distribution. The standard deviation of phase perturbations is $\sigma_{Ph} = \pi/2$ radians and std. for residual statics is $\sigma_T = 4\,ms$: (a) amplitude spectra; (b) phase spectra; (c) time-domain representation. The clean signal is shown by the blue line. Red and pink curves show numerically stacked quantities for two cases, whereas theoretical predictions are in black. All curves overlay each other on (b), confirming that stacking delivers a clean signal phase in all cases. Display (a) shows a good match between numerically stacked amplitude spectra (red, pink) with theoretical predictions in black. Figure (c) shows accurate arrival time (phase) but pronounced broadening of the wavelet due to combined actions of residual statics and phase perturbations.*