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Amplitude Radiation Pattern of a Virtual Source - Estimation and Correction

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SUMMARY

In the Virtual Source (VS) method we cross-correlate recordings at two receiver locations to create data as if one of these receivers is a Virtual Source and the other is a receiver. We study the amplitude radiation pattern of Virtual Sources. This pattern can be estimated by autocorrelation of the spatial Fourier transform of the downgoing wave field that is used for the VS creation in the special case of a laterally invariant medium. The generated VS data can be improved by deconvolution with the estimated amplitude radiation pattern in the FK-domain. The methodology is tested on a 1D elastic model, where it is shown that almost perfect amplitude retrieval is possible within a limited aperture of VS radiation. In general heterogeneous media the spatial Fourier transforms are not laterally invariant and the VS amplitude radiation pattern has to be estimated in a different way. We argue that Wigner distribution functions can serve this purpose. We show how these functions can be used for diagnosis of the spatial distributions of the wave fields that are used for VS creation, with a VS synthetic data example generated by the Peace River 2D elastic model.

Introduction

The Virtual Source Method is an innovative technique to image and monitor the subsurface in cases where complex overburden prevents seismics and VSP to deliver good results (Bakulin and Calvert, 2006). Placing receivers below complex overburden allows measuring the propagation response directly and apply time-reversal logic to redatum surface shots into downhole receiver locations without any additional information about the medium between source and receivers. Redatumed shots are called Virtual Sources. Theory suggests that a correct response can be recovered when sources are located on a closed surface surrounding receivers, however practical applications typically involve one-sided illumination with limited aperture (Bakulin & Calvert, 2006). This creates artifacts and in generally distorts Virtual Source amplitudes. An important improvement of the VS method was suggested by Mehta et al. (2007), who reasoned that separation of the up- and downgoing wave fields before cross-correlation would eliminate certain spurious events, caused by the lack of illumination from below the receiver array. In this paper we discuss the radiation characteristics of Virtual Sources in more detail. First, we show how the VS amplitude radiation pattern can be estimated and corrected in a laterally invariant medium using a horizontal array of receivers. Then we discuss some analogue ideas for laterally varying media. The theory is tested on 1D and 2D synthetic elastic datasets inspired by real fields in the Middle East and Canada.

Theory

In the current best practice of the Virtual Source (VS) method we cross-correlate the downgoing wave field $\hat{G}^{+}(x_{VS}, x_S, \omega)$ at VS location x_{VS} with the upgoing wave field $\hat{G}^{-}(x_R, x_S, \omega)$ at receiver location x_R and integrate over the source locations x_S (Mehta et al., 2007):

$$\hat{G}_{VS}^{-}(x_R, x_{VS}, \omega) = \int_{-\infty}^{+\infty} \left\{ \hat{G}^{+}(x_{VS}, x_S, \omega) \right\}^{*} \hat{G}^{-}(x_R, x_S, \omega) dx_S . \quad (1)$$

The result $\hat{G}_{VS}^{-}(x_R, x_{VS}, \omega)$ is supposed to converge to the reflection response between VS location x_{VS} and receiver x_R in an equivalent medium where the half space above the receiver array is replaced by a homogeneous medium. If the medium is laterally invariant, we can freely shift the data spatially and each Green's function can be synthesized from the central shot record. We apply spatial Fourier transformation to equation (1). With help of the shift theorem we can derive that the current practice of the VS method results in

$$\tilde{G}_{VS}^{-}(k, \omega) = \left\{ \tilde{G}^{+}(k, \omega) \right\}^{*} \tilde{G}^{-}(k, \omega), \quad (2)$$

where $\tilde{G}^{\pm}(k, \omega)$ is the FK-transform of the central shot record $\hat{G}^{\pm}(x, 0, \omega)$ and $\tilde{G}_{VS}^{-}(k, \omega)$ is the FK-transform of the central virtual shot record $\hat{G}_{VS}^{-}(x, 0, \omega)$. Recall that the VS data is supposed to converge to the reflection response of the medium below the receiver array $\tilde{R}^{+}(k, \omega)$. Alternatively we can state that the upgoing wave field is a convolution of the downgoing wave field with this reflection response, which can be expressed for a laterally invariant medium in the FK-domain as

$$\tilde{G}^{-}(k, \omega) = \tilde{R}^{+}(k, \omega) \tilde{G}^{+}(k, \omega) . \quad (3)$$

Substituting this representation into equation (2) we find that

$$\tilde{G}_{VS}^{-}(k,\omega) = \left|\tilde{G}^{+}(k,\omega)\right|^{2} \tilde{R}^{+}(k,\omega). \tag{4}$$

Note that the phase of VS data indeed represents the phase of the target reflection response. However, in terms of amplitudes we find the additional factor $\left|\tilde{G}^{+}(k,\omega)\right|^{2}$, which can be interpreted as the VS amplitude radiation pattern. It describes the imprint of the imperfect one-sided illumination at the receiver array and affects both the down- and upgoing wave fields that are used in the cross-correlation process. The perfect VS would be fed by a wave field with uniform spatial distribution as if $\left|\tilde{G}^{+}(k,\omega)\right|^{2}$ were replaced by unity. If this is not the case, the amplitude radiation pattern can be corrected by a stabilized deconvolution with the estimated amplitude radiation pattern in the FK-domain. For laterally varying media the previous reasoning needs to be modified. In this case the spatial distribution of the illuminating wave field depends on the location of observation, while the proposed spatial Fourier transforms only give an indication of the average distribution at the receiver array. However, we can make use of Wigner distribution functions, which can be interpreted as local Fourier transforms, to estimate the amplitude illumination pattern at a particular VS location. This estimate can then be used either for diagnostic purposes or to improve the radiation characteristics of the VS through a deconvolution correction in the FK-domain.

Example 1: 1D elastic model

We test our theory for laterally invariant media on a 1D elastic model (Mehta et al., 2007), being inspired by reservoirs as they typically occur in the Middle East. In this model, 321 sources are situated at 2 m depth with 5 m source spacing. The upper 200 m of the overburden consists of finely layered material. Below 200 m we find a homogeneous layer in which 161 receivers are situated at 250 m depth. Below the receivers we find four strong reflectors that we want to image. VS data is generated with equation (1). In analogy, the Ground Truth (GT) response is created by placing an active source at the VS location, with the half space above the receiver array replaced by a homogeneous medium. In Figure 1a we show the VS data generated with vertical force sources versus the GT response. In Figure 1b we show the same data after amplitude radiation correction through deconvolution in the FK-domain. Note better amplitude match after correction as well as a reduction of ringing and spurious behavior. If we use horizontal instead of vertical force sources the improvements are even more profound (Figures 2). The corrected radiation characteristics can be illustrated by showing the estimated amplitude radiation pattern in the FK-domain (Figure 3). Note that certain information cannot be recovered by the radiation correction, due to very weak or absent illumination. By combining horizontal and vertical forces the results can be improved (Figure 4a). If we mute the large angles of incidence for both VS and GT data, where illumination at the VS location is still insufficient, we find almost perfect convergence of VS and GT (Figure 4b). We have thus shown that we can construct a Virtual Source that radiates equal amplitudes within a limited aperture.

Example 2: 2D Elastic Model

Our second example stems from the Peace River 2D elastic model that was used by Bakulin and Calvert (2006). In this model 321 explosive sources are situated at 15 m depth with 5 m spacing. The receiver array consists of 81 geophones with 10 m spacing at 430 m depth. The upper 230 m of the subsurface consists of strong laterally varying heterogeneities, followed by a layered medium hosting the receivers and some horizontal reflectors. Figure 5a shows the generated VS data for the most central VS in the array in the FK-domain. As we can see there are clear spots of over- and under-illumination that can not be found in the equivalent GT data (Figure 5b). In Figure 5c we show the stack of FK-transforms of the tapered downgoing wave fields that were used in VS creation. Although this plot gives a fair indication about the average spatial distribution of the wave fields that feed the Virtual

Sources, they hold little information about the spatial distribution of the downgoing wave fields at a particular VS location. To obtain a more accurate local characterization we make use of Wigner distribution functions (Schleich, 2001), which can be interpreted as local Fourier transforms at the central VS location. In Figure 5d we have stacked Wigner distribution functions at the central VS location. Visual inspection shows similarities between this plot and the local VS data (Figure 5a). Wigner distribution functions can thus be used for diagnosis of the illumination at the VS location that influences the VS radiation characteristics, which can be done on a shot-to-shot or integrated basis. Eventually we can apply amplitude radiation correction by deconvolution of the VS data with the stack of Wigner distribution functions. Results of this operation are still tentative at the present stage of research.

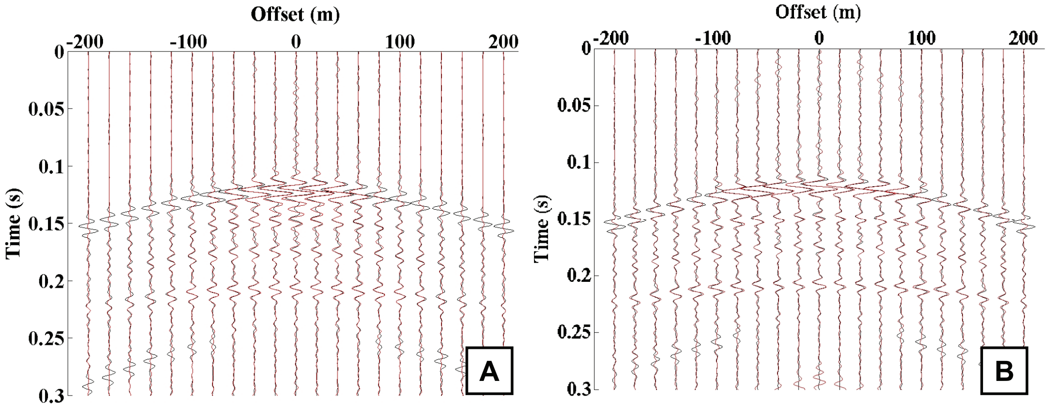

Figure 1: VS data generated by vertical force sources (red) vs. GT (black): **a)** before correction; **b)** after amplitude radiation correction.

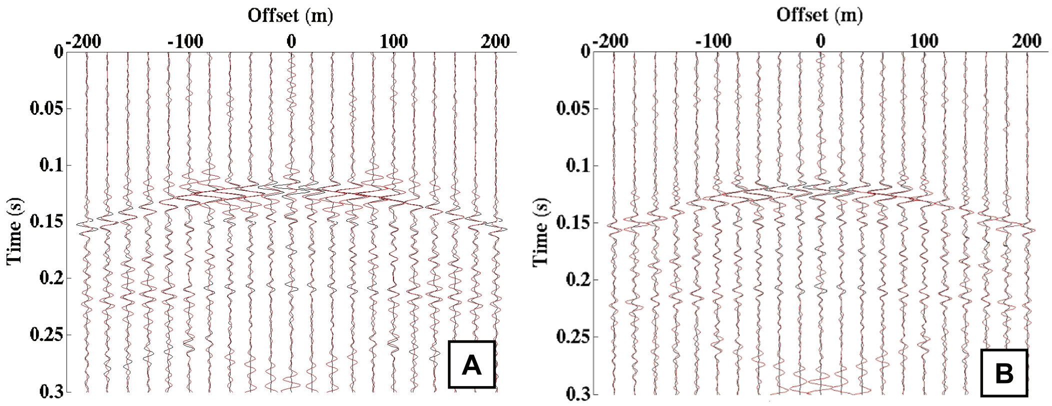

Figure 2: VS data generated by horizontal force sources (red) vs. GT (black): **a)** before correction; **b)** after amplitude radiation correction.

Conclusion

We showed that the amplitude radiation pattern of a Virtual Source (VS) in a laterally invariant medium can be estimated by auto-correlation of the downgoing wave fields that were used for VS creation in the FK-domain. VS data can be improved by deconvolution with the estimated amplitude radiation pattern in the FK-domain. Within a limited aperture of radiation we can create a Virtual Source with perfect radiation characteristics. For Virtual Sources generated in general heterogeneous media the spatial distributions of the illuminating wave fields at a particular VS location can be estimated with Wigner distribution functions of the downgoing wave fields, which can be interpreted as local Fourier transforms. These Wigner functions can be used either for diagnostic purposes or to manipulate the radiation characteristics of the Virtual Sources.

References

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Mehta, K., Bakulin, A., Sheiman, J., Calvert, R. and Snieder, R. [2007] Improving the Virtual Source method by wavefield separation, *Geophysics*, **72**, V79-V86.
Schleich, W.P. [2001] *Quantum optics in phase space*. Wiley-VCH, Berlin, Germany.

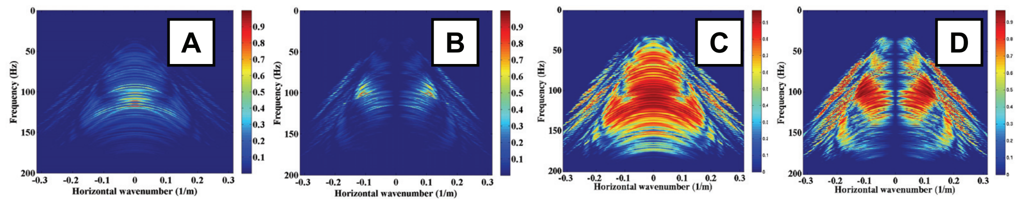

Figure 3: a) Estimated amplitude radiation pattern generated by vertical force sources;
b) Estimated amplitude radiation pattern generated by horizontal force sources;
c) Corrected amplitude radiation pattern generated by vertical force sources;
d) Corrected amplitude radiation pattern generated by horizontal force sources.

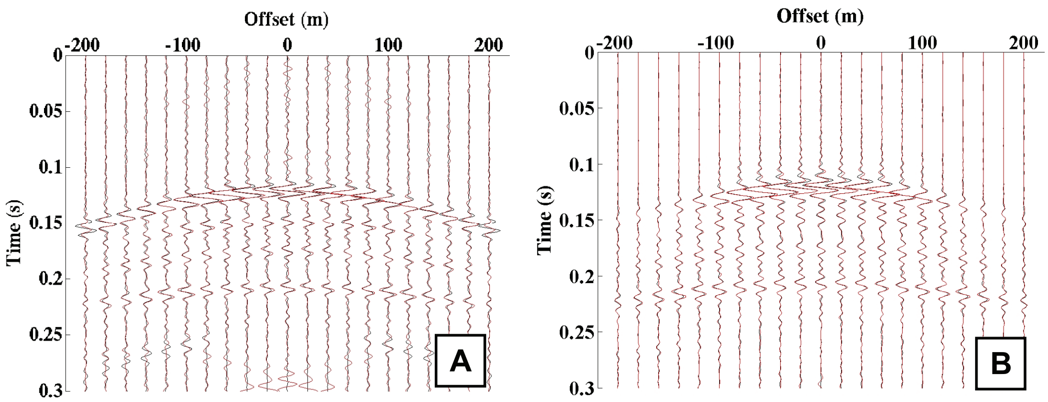

Figure 4: a) VS data generated by combined force sources after amplitude radiation correction (red) vs. GT (black).
b) same, with limited radiation aperture, enforced by additional FK-filtering.

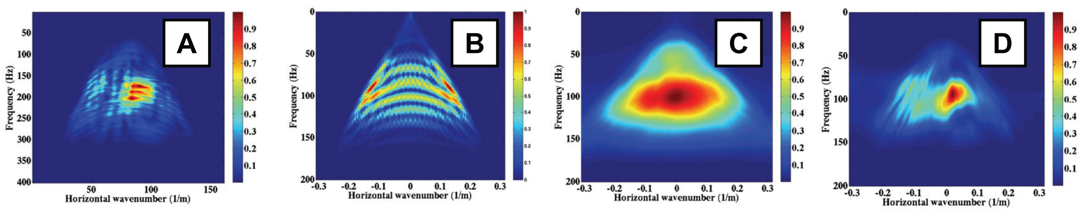

Figure 5: a) VS data in the FK domain (additional FK-filtering has been applied).
b): GT data in the FK domain (additional FK-filtering has been applied).
c): Stack of FK-transforms of the downgoing wave fields used for VS creation;
d): Stack of Wigner distribution functions of the downgoing wave fields used for VS creation.